Legal Metrology

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Abstract

A discussion on the Physics of Weights and Measures. The theory of electricity using six base units.

1 Introduction

Legal metrology is a series of definitions suitable to establish the working algebra for dimensional analysis, and for defining quantities. It does not handle the variations in nature. For example, the definition $\mathbf{B} = `\mu\text{'}\mathbf{H}$ does not account for either that in a magnet, μ might be a dyadic tensor, or even that B might not vary exactly with H at all. In the free-space condition, the definition remains true, and is suited for defining the quantity.

1.1 Some Conventions

Greek letters as prefixes are used in place of differential forms. Such might be regarded as part of the symbol, and serve to modify it.

β	gauss constant	$\beta = Q/\Phi$
γ	radiant constant	$D = Q/\gamma r^2$
ϵ	electric constant	permittivity
η	admittance of space	$\sqrt{\epsilon/\mu}$
κ	Linkage constant	$Q = I\kappa t$
λ	density over length	$\lambda Q = \mathrm{d}Q/\mathrm{d}l$
μ	magnetic constant	permeability
ϖ	symmetry constant	$\sqrt{\epsilon\mu}$
ξ	moment of quantity	$\xi Q = \int$ (coordinate) $\mathrm{d}q$
ϱ	density over space	$\lambda Q = \mathrm{d}Q/\mathrm{d}v$
σ	density over surface	$\sigma Q = \mathrm{d}Q/\mathrm{d}a$
τ	rate of change of Q	$\tau Q = \mathrm{d}Q/\mathrm{d}t$
'Q'	definition of Q	This equation defines the quantity Q
$1/\mathbf{R}^n$	unit vector	equal to $\mathbf{R}/R^{n+1}$

Equations involving vectors, do not replicate a unit vector in the numerator, since $1/\mathbf{R}$ and $\mathbf{R}$ produce a unit vector in the same direction. The equations then represent the true numerical relations.

Equations with quotes, thus $F = Q\text{`}\mathbf{E}\text{'}$ serve to define the quantity in quotes. Here the $\mathbf{E}$ is being defined, even though it does not occupy a side of the equation. This save setting the equation into a form it is not customary to see it in.

Chapter 1

Basic Concepts

2 Number

A number is something that is counted. Measurements are supposed to be a count of copies of some unit. However, while five cows is always that, five gallons might yield a different number when a different jug is used. Also, the measure less than a gallon, might not necessarily met in gallons, but by using a smaller jug, like a pint measure.

The supposition of measurements with just one number and one unit, usually expanded into decimals, is clearly not found in practice. For example, measurement in the English system are largely a series of numbers and units, such as miles and chains, or pounds and stones.

The truth is, people simply don't like fractions, and generally prefer a construction of several units, or a large number of a small unit. For example, people-heights are rated in centimetres or feet and inches, rather than metres or feet, with a fraction or decimal. A money amount like $1.50 might be said as 'a dollar fifty' or even 'a dollar fifty cents', rather than 'one-point-five dollars'.

Percentages (which might be expanded into 'basis points' or 'mills'), allow one to avoid decimal fractions, by rating the unit as a hundred or ten thousand, and the intermediate values as an integer. Sixty percent is the same as zero-point-six, but is more readily understood.

2.1 Cardinals and Ordinals

A cardinal number represents size: eg 'six'. An ordinal number represents position: eg 'sixth'. The **referrent** is the number represented by the cardinal or ordinal: eg, 'six'.

When one expresses a number or measure by cardinals, one gives the completed counts of each level, eg 'five hundred and sixty three', means 500 + 60 + 3. The tens-referent is '560', one says it is 3 after this. The hundreds-referrent is 500, meaning one has this number. The common number system is a cardinal structure, but such is rarely used of time, although the Mayans used it that way.

In an ordinal system, one refers to the order of the current layer. For example, in 563, one might say that it is in the sixth hundred, seventh decade and three. That means, for example, five hundreds have come and gone, and the sixth one is current. The referrents are then the *next* multiple of the place: ie 600, 570, 573. It is used of time in most places, because most people are forward looking.

For example, this is the twenty-first century, even though the year is 2014. The referrent here is twenty-one centuries (ie 2100), of which twenty have been wholly allocated, and the current one is the twenty-first. The months are numbered from 1 to 12, representing an ordinal count: a date in the first month means that no month has passed, and the current month is the first, has not ended. Likewise, the 18th day, means that seventeen days have gone and this is part of the eighteenth.

One gets some rather mixed meanings from people who do not appreciate this. Your sixtieth year *ends* on your sixtieth birthday. One gets it right with 'baby's first year', which ends when baby turns one. But by the time one gets to the other end of life, a person who is 87 years and 10 months of age, is in her 88th year, not her 87th.

Likewise, the 'eleventh hour', supposed to represent times just before noon (eg 11:59), actually represents times between 10:00 and 11:00. An ordinal referrent *ends* the period, not starts it. So the hour ending at 12:00 midday is the 'twelfth hour'.

The usual debate about 'when the century ends', is also simple: the structure is an ordinal, so the 20th century ended at the change from 1999.11.30 to 2000.00.00 in the Mayan style. In the rest of the world, one would change from 1999.12.31 (ie completed the twelfth month and the 31st day), to beginning a whole new package: the first day of the first month of the first year that the next round point in 00.00.00 is 2100.00.00.

2.2 Multiples and Divisions

The great diversity of fraction-schemes against the simple notion of counting, and the lateness of the appearance of the decimal numbers, might call into mind what is going on here.

The multiples and divisions are handled in different parts of the brain. Butterworth[But99] gives accounts of various patients whose brain has been damaged in different parts, and have lost the facility of one, without the other function. Also, one is lead to conclude that while only one part of the brain is used to make multiples, many different formations are used for making fractions.

Multiples have a very simple construction. Twenty units make a score, and one has a number of scores, and a number of units. The number of scores is then counted, and the process is repeated until there is only remainders. A number is then a series of remainders, eg 2014 becomes 100 score and 14, and 100 becomes 5 score and 0. In turn, 5 becomes 0 score and 5, one has then divided 2014 into remainders against 20 as $\frac{5}{20}\frac{0}{20}\frac{14}{20}$;

The division measures centre around weight, particularly bullion-weight.

Egyptian The most enigmatic fraction system is that of the egyptians, who preferred to use a series of unit fractions, which we might write $1+\frac{1}{a}+\frac{1}{b}$ in a style closer, of the form 1 'a 'b. The multiplication tables such have come to us are reductions of 'a 'a into different numbers. For example, '3 '3 might be written as '2 '6. Gillings[Gil82] gives an authoritive account of the arithmetic associated with these numbers. They were certainly used well into the time of the Roman empire, where '3 '8 is used for 5 uncia 8 drams.

Sumerian The Sumerian system for weights is to divide a measure of three-score[Neu69] into alternating measures of six and ten. A weight of a talent, is divided into 60 mina, of 60 shekels, of 60 berah, of 3 barleycorns. In effect, one uses the division of weights as a kind of fraction. It spread into astronomy, because its toolkit was more extensive than other number systems in use, and has come to us in that form. The actual calendar and angle system are a later mishmash.

Greek Greek fraction is to divide the measures into smaller grains, and give a ratio of grains. A fraction like '3 '8 might be written as 11 parts, where 24 make the whole. Such fractions were used by the Mayans too.

Roman Roman fractions are by weight, where the unit is understood. The foot, pound, and hour, are divided into 12 uncia, each of 8 drachms of 3 scruples, and so forth. A division into 16 digits, in the manner of the foot is also known. The uncia is also divided, originally into six solidus[1]., each of 24 carats[2]. Such fractions are still in use.

Latin Latin fractions are added fractions, which we deal with in the next subsection. In modern terms, it might be regarded as a series of continued numerators. A foot is divided into 12 inches, and an inch is divided as if it were the unit, to 12 lines into a binary fraction. One can write added fractions as a series of adjacent fractions, like $3\frac{3}{12}\frac{3}{8}$. Fibonacci used this kind of fraction in his *Liber Aceri.*

Radix A radix-fraction is a sub-multiple. The idea here is that if you divide a pound into smaller measures, one should count the smaller measures, and a column would make a pound. Such divisions do happen[3]. Radix-fractions first appear in the form of using primes, seconds, and thirds,

[1]This unit survived longer in the Moorish lands as a *metkal.* It's about the size of a shilling coin.

[2]A carat represents a carob-seed, which has a weight of about 200±10 milligrams. This weight is normalised in different systems, and eventually 'internationalised' to 205 mg, and then 200 mg. A carat also represents a 24th part of measures: the Turkish ell was divided into 24 carats, and the carat as a 24th pound, represents a weight required to give 24 carats to the solidus.

[3]A greek *mina* is divided into 100 drachma of 6 obols. A count of 250 drachma makes then 2 mina and 50 drachma. The romans absorbed the drachma, but fitted it into their division system, so a drachma is an eighthþof an ounce.

as far as needed. The C.G.S. fraction system is based on this kind of division, such as the Ångström being a tenth-metre, that is, the tenth decimal division of the metre. The radix-point separating units from the primes, seconds, etc, is a later division.

2.3 Added Fractions

The prototype of measure is a series of nested or added fractions, each subsequent fraction being a division of the previous measure. A stone is divided into fourteen pounds, and a pound into sixteen ounces. A weight given by example, as 19 st, 11 lb, 12 oz, has no single unit or number. One could write this kind of fraction in the style of added fractions, being $19\frac{11}{14}\frac{12}{16}$ stones.

One does not have to suppose that the added fraction has a root[4] immediately after the first number. One could as readily suppose $19\frac{11}{14};\frac{12}{16}$ pounds, or a similar construction of ounces. Such measures are *superdivisions*.

2.4 Numerals and Zero

A system of numerals allows one to distribute stones on a number-board, representing the series of remainders of the count. One needs to know for each symbol, of where to place one or more stones, either relatively or absolutely. Many cultures still use a diverse range of symbols, and even the spoken language differs from the numeric forms.

Systems based on tokens equate to placing a stone in a particular column. For example, the symbol M corresponds to placing one stone in the thousands column, while a C places it in the hundreds column. Such a notation, written on the stones themselves, might allow the stones to be placed in a bag, and correctly unpacked. Such a modern example includes coins, where an amount can be extracted from a purse and spread correctly on a table.

A different system uses a series of remainder-staves, which are ordered from the last (highest) to the first. These remainder-staves are usually called *digits*[5]. Note that digits are not the sort of thing one can bag: the number CIII might well be bagged, but the order of digits '103' needs to be kept. Stones with '1', '0' and '3' by themselves might be arranged in several different ways. The modern digits descend from the subcontinent, through the moors.

A third system consists of a pair of numbers, a remainder-staff and a column-staff. Such might be rendered as ligatures. Instead of writing 103, one writes, eg 1C 3I, meaning, one in the hundreds column, and three in the units column. Such a system might be used in stamps, where an amount of $18.50 is made of a ten-dollar stamp, an eight-dollar stamp, and a fifty-cent stamp. Such a system is used in the far east, and english numbers[6] were in the style of iii C v for what we write 365.

The Sumerian system is a series of deal-staves. The first two staves represent a number, while subsequent staves represent the numerator of added fractions, of alternating divisors of six and ten. The staves for the six-divisors are written as horizontal wedges, while the ten-staves are vertical staves. One might in a more modern context, use the letters A to E to represent 1/6 to 5/6. The digits 1 to 9 serve for the ten-fractions 1/10 to 9/10. One would then write decimal 15 as 'A5'. A number like 1C is not 90, but 1 3/6.

Zero, then has multiple meanings. The word comes from 'empty', and is usually read as an empty column. But if one is placing the columns in the symbol, there is no need to represent an empty spot. The roman CIII does quite nicely without the need to represent no element of the tens-column. It's only really needed when one is giving the column-content in sequence.

The Egyptians certainly did understand zero, and had a rune for it. But it is not needed to show that one has no ten-cent pieces, rather that one had no money at all! That is, zero represents the number we write as '0'.

When one has a position-string notation, a zero or some device is needed to show empty medial columns, such as the 0 in 103, along when the root column itself is empty. Such is needed, for example to show 1030. Because we know in the sumerian system, such outside-spaces appear to make the number smaller (ie 005 is 0.05), the system is a division system, such that 1C3B lies between 1C3 and 1C4. 1C is 1 1/2 but 01C is 1/40, that is, 1/60 of 1C.

[4] Latin *radix* means root.

[5] Digit is from the latin for finger or toe. A foot consists of sixteen digits.

[6] English numbers, until the plague of digits, were written at M=1200, C=120.

A different kind of zero might be used to show that some medial unit is zero, while some measures on each side are non-zero. It's like writing £5 - 0 s $3^{1}/_{2}$ d. It's sometimes written as a dash, especially if the data is in columns. A similar example is like writing a dash or pair of dashes after a neat number of dollars (eg $18—).

Setting zeros on the off-side of the number, such as 0103, or 0.1030, does not change the size of the number. Instead, its presence might be taken to mean a four-digit number is expected, or that the error in the quoted value is a unit in the last place (ie .1029-.1031 rather than .102 to .104).

A 'semi-medial zero' is one where while no column is empty, the columns consist of rows, and there's a full column's worth of cells missing. The sumerian number for 11 is A1, while the one for 601 is A 1. Here both the unit of the 60's column, and the tens of the 1 are absent, yet they are on different columns, eg $\genfrac{}{}{0pt}{}{1}{0}, \genfrac{}{}{0pt}{}{0}{1}$ against $\genfrac{}{}{0pt}{}{1}{1}$

Chapter 2

Electromagnetism

Legal metrology provides the base units for a system of coherent units. For example, the definition of a square foot as the area of a square of side one foot, does not imply that it is 144 square inches.

3 Electromagnetism with six dimensions

The great diversity of systems in electromagnetism might be correctly read if one supposes six base units, some of which are converted into numeric constants. But comparing the resulting equations allows us to reconstruct these constants.

One can simply restore these constants. The relation of flux Ψ and the enclosed charge Q is in SI $\Psi = Q$ and C.G.S. $\Psi = 4\pi Q$, to get a single equation $\Psi = QS$. One can even read S as a solid angle. Using equations like those that Heaviside extracted from Maxwell's work, one can even prove that two such constants suffice. This is the approach followed by Leo Young[You69]. Such an approach might well work for those who have to deal with two distinct systems, but it offers little explanation on how such might have risen.

The following equations mean to give both a logical source for the quantities, and a definition that we might apply six-dimensional analysis to.

3.1 Radiant Fields

Like light and gravity, electrics and magnetics might might be treated as a *radiant field*. What this means is that the source radiates flux-particles, which travel outwards. The number does not change, but the surface area increases as the square of radius. Such is the common inverse square law.

Light and heat are *scalar* fields, which means that as flux strikes a thing, the load is off-loaded, but no hint of direction is passed. Gravity, electricity and magnetism are *vector* fluxes, which means the off-load also moves the destination towards or away from the source. Sources are *charges*, the effect of flux is *field*.

	Electricity	Magnetism	
Charge	$\mathbf{F}$ = 'Q'$q/\gamma\epsilon\mathbf{R}^2$	$\mathbf{F}$ = 'P'$p/\gamma\mu\mathbf{R}^2$	$\mathbf{R}$ is radius-vector
Field	$\mathbf{F}$ = '$\mathbf{E}$'Q	$\mathbf{F}$ = '$\mathbf{H}$'P	
Potential	$\mathbf{E} = \nabla$'V'	$\mathbf{H} = \nabla$'U'	∇ = grad
Capacitance	V = 'C'Q		
Flux density	'$\mathbf{D}$' $= Q/\gamma\mathbf{R}^2$	'$\mathbf{B}$' $= P/\gamma\mathbf{R}^2$	
Flux	'Ψ' $= \mathbf{D}\cdot\mathbf{a}$	'Φ' $= \mathbf{B}\cdot\mathbf{a}$	$\mathbf{a}$ is area normal
Perm'ity	$\mathbf{D}$ = 'ϵ'$\mathbf{E}$	$\mathbf{B}$ = 'μ'$\mathbf{H}$	permittivity, permeability

One can substitute into the last equations, the definitions of $\mathbf{D}$ and $\mathbf{E}$ to get the first equation. Although it appears that one equation is redundant, the first equation creates a concept of charge, governed by an inverse-square law, and an arbitrary constant. This constant is partly decomposed in the following definitions.

3.2 Dipoles and Polarisation

A *moment* of X is the integral over the elements of X, of some the coordinate of X and its element. Specifically, $\xi X = \int \mathbf{r} \mathrm{d}x$. When there is a net value of X, the moment changes with coordinate. However, it is possible to remove this dependence.

Momentum is the rate of change of moment, or $\tau\xi M$. Because the coordinate system is not changing too, one only sees an intrinsic measurement of a moving object, specifically $\tau\xi M = M\mathbf{v}$.

Another coordinate-free moment happens when the integral over X is zero, ie $\int \mathrm{d}x = 0$. While the total sum of quantity X might be zero, there is a spacial distribution, and this produces an intrinsic vector.

	Electricity	Magnetism	
Charge dipole	‘$\mathbf{p}$’ $= \xi Q$	‘$\mathbf{j}$’ $= \xi P$	moment of charge
Polarisation	‘$\mathbf{P}$’ $= \varrho\mathbf{p}$	‘$\mathbf{J}$’ $= \varrho\mathbf{j}$	ϱ is space-density
e/m ic dipole	$\mathbf{T} = \mathbf{D} \times$ ‘$\mathbf{k}$’	$\mathbf{T} = \mathbf{B} \times$ ‘$\mathbf{m}$’	‘T’ $= \xi F$ is Torque.
e/m isation	‘$\mathbf{K}$’ $= \varrho\mathbf{k}$	‘$\mathbf{M}$’ $= \varrho\mathbf{m}$	electrisation, magnetisation
Susceptibility	$\mathbf{K} =$ ‘χ_e’$\mathbf{E}$	$\mathbf{M} =$ ‘χ_m’$\mathbf{H}$	

The vector $\mathbf{K}$ ‘electric potential’ is an example of a division of a quantity. This is divided from $\mathbf{E}_i$ in the way $\mathbf{H}_i$ gives $\mathbf{M}$. $\mathbf{K}$ was formerly used as an alternate symbol for the vector we call $\mathbf{E}$. The partition also causes problems when the equations like $\mathbf{H} = \mathbf{B}$, due to $\mu = 1$.

Although natural dipoles exist, the presence of a field can cause a movement or *displacement* of charges in the material, which serves to reduce the field inside the body. However, the induced polarisation is fixed by an intrinsic property of the material, being the susceptibility.

The *induced* fields are proportional to the torque-producing densities as defined above by the following relations. The constant β in the following equations arise from geometry. An induced charge-dipole $\mathbf{P}$ is a direct measure of the induced flux density which is causing it. It is this that makes the vector $\mathbf{D}$ come to be called displacement.

$\mathbf{P} =$ ‘β’$\mathbf{D}_i$ $\qquad$ $\mathbf{J} = \beta\mathbf{B}_i$ $\qquad$ $\mathbf{K} = \beta\mathbf{D}_i$ $\qquad$ $\mathbf{M} = \beta\mathbf{B}_i$

Because susceptibility is defined as a ratio of vectors, the derived relation $\epsilon_r = 1 + \chi_e/\beta$, gives where $\gamma = 1$, then $\epsilon_r = 1 + 4\pi\chi_e$, and where $\beta = 1$, then $\epsilon_r = 1 + \chi_e$.

The relation $\gamma = 4\pi\beta$ derives from Gauss's flux law. Specifically, the net flux through a surface is directly proportional to the charge enclosed in the surface. A very thin surface on the surface of a dielectric medium gives the charge $P \cdot \mathbf{A}$, against the reduction in the flux, $D \cdot \mathbf{A}$. In the simple case, the flux is simply $D \cdot A$. The enclosed charge is derived from the surface density and area $\sigma Q \cdot A$.

One can find from an enclosed charge Q and the flux through the surface, Ψ, that $Q = \beta\Psi$. For a point charge q, one might take a sphere of radius R. The surface is then $A = 4\pi R^2$. the flux density is $\mathbf{D} = Q/\gamma\mathbf{R}^2$. Because the vector area is parallel to D, the total flux through this surface is $\Psi = \mathbf{D} \cdot \mathbf{A}$, or $\Psi = 4\pi Q/\gamma = Q/\beta$, whence $\gamma = 4\pi\beta$.

Some systems set $\gamma = 1$, leaving $\beta = 1/4\pi$. More recently, systems follow Heaviside's advice and set $\beta = 1$, making $\gamma = 4\pi$.

3.3 The Electromagnetic Link constant κ

Electrics and magnetics are linked by way of the electromagnet. Specifically, the model of magnetic charge fails to explain how an electromagnet works, and far from radiating from one source to another, magnetic flux forms loops. An ordinary magnet appears as a magnetic dipole, because the inner parts of the magnet can't be reached.

A magnetic dipole of the measure m, can be produced by an electrical current I flowing around a vector-area $\mathbf{a}$, by the new relation $\mathbf{m} =$ 'I'$\mathbf{A}$. The current is flowing in a closed loop. Where A is a surface bounded by the closed loop, the integral $A = \int \mathbf{n} da$, where n is a unit vector vertical to the element da[1].

The current in the loop is called Ampere's current. A similar induction arises by way of faraday's induction by way of a potential produced by a changing flux.

	Electricity	Magnetism	
Current	'i' $= \tau Q$	$\mathbf{m} =$ 'I'$\mathbf{a}$	ampere current
Potential	$\mathbf{E} = \nabla$'V'	'v' $= \tau\Phi$	faraday potential
Power	$\tau W = V.i$	$\tau W = v.I$	(equal)
E-M Linking	$i = I$'κ'	$v = V\kappa$	$\tau W = iv/\kappa = IV\kappa$

By Leo Young, $\kappa = 1/U$, where U takes the form of the unit 'turn', as in 'ampere-turn'. In effect, a current of i flowing through a coil of t turns produces an ampere-current of I is equal to i current by t turns. Likewise, a real voltage V is manifest by a faraday potential v through several turns. But 'volt-turn' is rarely heard.

A distinction is to be made between the Sommerfeld and Kennelley magnetic dipoles. The Sommerfeld system is the one widely used, but it is not impossible to have both included in the set of quantities. One associates magnetic dipole with either $\mathbf{J}$ or $\mathbf{M}$. One just has to be wary of reading texts.

	Kennelley	Sommerfield
Dipole	$j = \mu I\mathbf{a}$	$m = I\mathbf{a}$

A similar distinction across electricity gives Ψ' measured in Volt metres. It is rare to be met, but in Alan Phillips's Electromagnetism (the text the author was taught through), the unit of flux is $\mathbf{E} \cdot \mathbf{a}$ rather than the more common $\mathbf{D} \cdot \mathbf{a}$.

3.4 The new Electromagnetic System

The definitions by magnetic charge no longer work. Instead, one has to devise a new set of definitions. The following table shows the definitions of the charge, field, and flux-like elements in electrostatics, magnetostatics, and electromagnetics.

	Electrostatics	Magnetostatics	Electromagnetics
'Charge'	$\mathbf{F} =$ 'Q'$q/\gamma\epsilon\mathbf{R}^2$	$\mathbf{F} =$ 'P'$p/\gamma\mu\mathbf{R}^2$	$\lambda\mathbf{F} = 2\mu$'I'$i/\gamma\mathbf{R}$
constant	'K_c' $= 1/\gamma\epsilon$	'K_m' $= 1/\gamma\mu$	'K_a' $= \mu/\gamma$
'Field'	$\mathbf{F} = Q$'$\mathbf{E}$'	$\mathbf{F} = P$'$\mathbf{H}$'	$\lambda\mathbf{F} =$ '$\mathbf{B}$' $\times \mathbf{I}$
'Flux'	'$\mathbf{D}$' $= Q/\gamma\mathbf{R}^2$	'$\mathbf{B}$' $= P/\gamma\mathbf{R}^2$	'$\mathbf{H}$' $= 2I/\gamma\mathbf{R}$
perm'ity	$\mathbf{D} =$ 'ϵ'$\mathbf{E}$	$\mathbf{B} =$ 'μ'$\mathbf{H}$	$\mathbf{H} = \mathbf{B}/$'μ'
Potential	$\mathbf{E} = \nabla$'V'	$\mathbf{H} = \nabla$'U'	$\mathbf{B} = \nabla \times$ '$\mathbf{A}$'
Electrokinetic Potential			'$\mathbf{E}_k$' $= \tau\mathbf{A}$

The equation $\lambda\mathbf{F} = 2\mu$'I'$i/\gamma\mathbf{R}$ is used to define current, particularly in the form $\lambda\mathbf{F} = 2K_a Ii/\mathbf{R}$. It is this equation, and a similar one that we shall derive, which forms the basis of the electromagnetic velocity constant.

[1] If one imagines the loop to be covered with a second surface, the total surface is then a solid, which has a volume. Since volume is calculated as the moment of vector-surface (ie $V = \xi \cdot \mathbf{A}$), and volume is independent of coordinate, the vector-sum of a closed surface $\mathbf{A} = \int \mathbf{da}$ must be zero. Dividing the surface into a fixed part, and a variable part, the size of the two parts must be of equal magnitude and opposite sign. Thus the variable part has a constant vector area.

3.5 The Eight Vectors

A brief table showing the dimensional placing of the quantities is in order. In stead of using the standard LMTQ dimensions, which is prone to produce square roots and divisions, the values of EL do not. In order to do this, we need to suppose that the electric and magnetic constants are the product and quotient of two new constants.

$$\text{`}\varpi\text{'} = \sqrt{\epsilon\mu} \qquad \text{`}\eta\text{'} = \sqrt{\epsilon/\mu} \qquad \epsilon = \varpi\eta \qquad \mu = \varpi/\eta$$

Since $\gamma = 4\pi\beta$, we suppose these have the same dimension too.[2]. The equation for E is $E = Q/\gamma\epsilon R^2$, or dimensionally, $E = Q/L^2\beta\varpi\eta$. Since we mean to eliminate divisions, this can be written as $Q = EL^2\beta\eta\varpi$. Force is $EQ = E^2L^2\beta\eta\varpi$. One can solve for magnetic charge $P = Q_m$, as $Q_m = EL^2\beta\varpi$.

The arrangement across the table are by length operators: ∇, ξ, and the various density parameters λ, σ and ϱ. The remaining variables appear in the first column. L^0 contains eight vectors.

The first four are extrinsic field-like vectors. The group contains fields, fluxes, and potentials. The second group of four are intrinsic dipole vectors, the group has charges and charge-distributions and flows.

	L^0	L^1	L^2	L^3
E	$\mathbf{E}$	V		
$E\eta$	$\mathbf{H}$	U		
$E\eta\varpi$	$\mathbf{D}$		Ψ	
$E\varpi$	$\mathbf{B}$	$\mathbf{A}$	Φ	
$E\beta\eta\varpi$	$\mathbf{P}$		Q	$\mathbf{p}$
$E\beta\varpi$	$\mathbf{J}$		Q_m	$\mathbf{j}$
$E\beta$	$\mathbf{K}$			$\mathbf{k}$
$E\beta\eta$	$\mathbf{M}$	I	p	$\mathbf{m}$
$E^2\beta\eta\varpi$	$\mathbf{w}$		$\mathbf{F}$	$\mathbf{T}$, W

3.6 The Rule of Substance

The process of *rationalising* formulae involves shifting a factor 4π around. Since this is not in the standard dimensional analysis, one can not use that form. Instead, one needs to extend the dimensions to let this happen. We have done this here.

The table above shows the bulk of the electromagnetic quantities, in three groups. The first group contains *fields*, *fluxes* and *potentials*, and their densities. None of these represent actual substance, and all of these have no factor in β.

The second group contains charges, dipoles, polarisations, their densities and moments. All of these in the presence of a field, might produce force or torque, and are therefore a kind of substance. Note that magnetic charge is a substance were it to exist. All of these have a factor of β.

The third group contains the mechanical quantities with a quantity in M. These all represent substances, their densities and moments, and likewise contain a factor β.

β^{-1}	K_c	K_a	K_m	R, Z	L
β^0	ϵ	μ	Z_w		
β^1	χ_e	χ_m		G	Cap

[2] When area is no longer considered as a direct measure of L^2, these have different dimensions, that $\gamma L^2 = \beta$ area. But we suppose here that area is L^2

4 Electromagnetic Theory

No new quantities are introduced in this section. Instead, it is a discussion on the unification of electrics, magnetics, and light from separate fields into a unified theory.

4.1 The Electromagnetic Velocity Constant κ/ϖ

The measure κ is a direct connection between electricity and magnetism.

One can measure this by experiment, by measuring the same effect by electrical means, and then by magnetic means. It can also be set by theory. Wilhelm Weber and Rudolf Kohlrausch first measured this at np[mm/s]3.107e10.

The first case to consider is Ampere's law, giving $\lambda\mathbf{F} = 2\varpi\text{‘}I\text{’}i/\gamma\eta\mathbf{R}$. This is Ampere's law, with $\mu = \varpi/\eta$.

The second case is two parallel wires, charged to λQ. The value sought is so that the force per length λF is the same size. The way to find the force is to find the flux, of the first wire at the second wire. A cylinder centred on the first wire, of radius R and length L is used. No net flux flows over the end pieces, so the total flux is $\Psi = \lambda QL$. The flux density is $D = \sigma\Psi = \lambda QL/2\pi\beta LR$. Since $\lambda F = E\lambda Q$, one finds directly

$$\lambda F = \lambda Q\lambda Q/2\pi\beta\epsilon R$$

One now sets the forces equal, and evaluates $\gamma = 4\pi\beta$ to get:

$$\lambda Q\lambda Q/2\pi\beta\epsilon R = \mu Ii/2\pi\beta R$$

$$(\lambda Q)^2/\varpi\eta = \varpi I^2/\eta$$

Because I is a magnetic quantity, one must convert it to electric by way of $I = i/\kappa = q/\kappa t$. Likewise, λQ might be written q/l. If q is set equal, then l and t vary, such that for example, λQ is not 0.001 Vb/ft, but 1 Vb/1000 ft. Likewise, t is the time a current takes to deliver 1 Vb, eg 1000 Vb/sec = 1 Vb / 0.001 sec. This allows us to eliminate q.

$$(q/l)^2/\varpi = \varpi(q/\kappa t)^2$$

$$q^2/l^2 = \varpi^2 q^2/\kappa^2 t^2$$

$$l/t = \kappa/\varpi = \text{‘}v_{em}\text{’}$$

Since $\varpi^2 = \epsilon\mu$, one can write $\epsilon\mu v_{em}^2 = \kappa^2$. Weber and Karlsraus measured the value of v_{em} from electrical and magnetic fields.

4.2 Maxwell's Equation

The modern way of defining electricity is a similar flux/field approach, but uses point vector functions, rather than large-scale rules. The usual statement of Maxwell's equations are those of flux, and Faraday's and Ampere's law. These are shown in bold face below. But these by themselves can not define electromagnetism, and the other equations are usually inserted by way of aside.

	Electric	Magnetic	
field	$\mathbf{F} = Q\mathbf{E}$	$\lambda\mathbf{F} = \mathbf{I}\times\mathbf{B}/\kappa$	
Perm'ity	$\mathbf{D} = \epsilon\mathbf{E}$	$\mathbf{B} = \mu\mathbf{H}$	
flux	$\nabla\cdot\mathbf{D} = \varrho Q/\beta$	$\nabla\cdot\mathbf{B} = 0$	
Faraday Law	$\kappa\nabla\times\mathbf{E} + \tau\mathbf{B} = 0$		
Ampere law	$\kappa\nabla\times\mathbf{H} - \tau\mathbf{D} = \mathbf{J}/\beta$		$\mathbf{J} = \varrho\mathbf{v}Q$

The start of the push for 'rationalisation' began with authors who started with equations like this. Oliver Heaviside was the earliest recorded. In essence, one sets $\beta = 1$ and thus $\gamma = 4\pi$. One does not completely banish the 'eruption of 4π'. but in a rationalised system, one finds 4π connected to spherical symmetry, and 2π to circular or cylindrical symmetry.[3]

4.3 Electromagnetic Waves

The solution to Maxwell's equations in free space is that of a wave. One first removes **J** and ϱQ from the equations.

$$\nabla \cdot \mathbf{D} = 0 \qquad \kappa\nabla \times \mathbf{H} = \tau\mathbf{D}$$

Using the perm'ity equations, one reduces Faraday's and Ampere's law to.

$$\kappa\nabla \times \mathbf{E} = -\mu\tau\mathbf{H} \qquad \kappa\nabla \times \mathbf{H} = \epsilon\tau\mathbf{E}$$
$$\kappa(\kappa\nabla\times)\nabla \times \mathbf{E} = -\mu\tau(\kappa\nabla\times)\mathbf{H} \qquad \kappa(\kappa\nabla\times)\nabla \times \mathbf{H} = \mu\tau(\kappa\nabla\times)\mathbf{E}$$
$$\kappa^2(\nabla \times \nabla \times \mathbf{E}) = \mu\epsilon\tau^2\mathbf{E} \qquad \kappa^2(\nabla \times \nabla \times \mathbf{H}) = \mu\epsilon\tau^2\mathbf{H}$$

The double-curl $(\nabla \times \nabla\times$ simplifies to $\nabla(\nabla \cdot \mathbf{V}) - \nabla^2\mathbf{V}$ The first term is a zero-vector, so the equations simplify to a wave equation. The speed of the wave is found by moving all the constants to the left-hand-side.

$$\kappa^2\nabla^2\mathbf{E} = \mu\epsilon\tau^2\mathbf{E} \qquad \kappa^2\nabla^2\mathbf{H} = \mu\epsilon\tau^2\mathbf{H}$$

This gives $c^2 = \kappa^2/\mu\epsilon$ or $\varpi = \kappa/c$.

Maxwell compared the electromagnetic velocity constant as measured by Weber and Kohlrausch, and compared this with the value of light as measured by Fizeau, $3{,}15 \cdot 10^{10}$ mm/s, and concluded that light and electromagnetic waves travel in the same medium (ether).

Henreich Hertz conducted experiments with rotating magnets, which would produce undeniable electromagnetic waves, to show that electromagnetic waves have the properties of light. It is also possible to show that properties of light, such as Snell's law of refraction, are reflected exactly from electromagnetism.

4.4 Poynting's Vector

The flow of energy in electromagnetic fields is given by a vector found by Poynting and by Heaviside. It can be derived from Maxwell's equations as follows:

$$\kappa\nabla \times \mathbf{E} = -\tau\mathbf{B} \qquad \kappa\nabla \times \mathbf{H} = J + \tau\mathbf{D}$$

Taking the dot-product of H over the first, and E over the second, gives

$$\kappa\mathbf{H} \cdot (\nabla \times E) = -\mathbf{H} \cdot \tau\mathbf{B} \qquad \kappa\mathbf{E} \cdot (\nabla \times H) = -\mathbf{E} \cdot \tau\mathbf{J} + \mathbf{E} \cdot \mathbf{J}$$

The difference between these equations gives $\mathbf{H} \cdot (\nabla \times E) - \mathbf{E} \cdot (\nabla \times H) = \nabla \cdot (\mathbf{E} \times \mathbf{H})$.

4.5 Snell's Law

5 Bohr's Atom

Bohr's model of the atom shows the interface between quantum and classical physics. It is a very good approximation of the Hydrogen atom, and similar ions with one electron. Many of the modern constants can be understood by way of a discussion of this model. The model does not invoke relativity.

One should note that the failings of the model is one of the reasons that the quantum theory has moved onto more advanced theories including relativity.

Here, we write $\epsilon = \eta\kappa/c$, and $\mu = \kappa/\eta c$.

[3] One finds that the earliest attempts to write gravity in rationalised forms also follow the construction of Maxwell-like equations.

1.	F	$= c\,Ze\,e\,/\gamma\eta\kappa r^2$	Electrostatic field
2.	F	$= m\omega^2 r = mv^2/r$	Centripedial or Hubswingth
3.	E	$= hf = hc/\lambda$	Planck's constant
4.	$N\,h$	$= 2\pi\, mvr$	Bohr's quantisation

N is a quantum number, takes the form of an integer.
The equality of forces gives

	$Z\,e^2 c/\gamma\eta\kappa$	$= mv^2 r$	
5.		$= N\,hv/2\pi$	Bohr's quantisation (4.)
6.		$= N^2\,h^2/4\pi mr$	Second application.

Somerfeld's Fine Structure Constant is now defined from Equation 5.

7.	v/c	$= 2\pi Ze^2/N\,h\gamma\eta\kappa$	Equation 5.
		$= (Z/N)\,e^2/2\beta\eta\kappa h$	
8.	$\mathtt{k} = 1/\alpha$	$= 2\beta\eta\kappa h/e^2$	Fine structure hundred = 137.036
		$= \gamma\eta\kappa\hbar/e^2$	in gamma-form.
9.	v	$= Z/N\,c/\mathtt{k}$	Atoms limited to $Z > \mathtt{k}$.

The value $\alpha = 1/\mathtt{k}$ is Somerfeld's **Fine Structure Constant**, with allusions to the fine structure of spectra.

The implication of (9) above, is that electrons in an element Z=137 would be moving near light speed. When all things are taken to account, the value which can permit $N = 1$ (ie 1s orbitals), is Z=173.

The radius of the atom is found from equation (6.), as

	r	$= N^2 h^2 \gamma\eta\kappa/4\pi^2 Ze^2 mc^2$	
		$= (N^2/Z)\,\beta\eta\kappa h^2/4\pi^2 e^2 mc$	
10.	r	$= (N^2/Z)\,\mathtt{k}h/2\pi mc$	Bohr radius when $N = Z = 1$
11.	vr	$= (N)h/2\pi m$	Quantum of circulation

The Bohr magneton is defined in terms of an electron in the first orbit. The relation is $m = I{\cdot}a/kappa$. The current at a point is made up of the same electron passing it many times, that is $I = ev/(2\pi r)$. The ares is circular, ie πr^2, and $\mu = \kappa/\eta c$.

The Gyromagnetic ratio gives a frequency of procession, when a magnetic dipole is placed in a field **B**.

The magnetons are more exactly known in terms of the 'magnetic mass' as defined below, then in terms of the magnetons. In practice, one describes these in terms of bohr or nuclear magnetons.

The Zeeman effect is a splitting of lines, when a magnetic moment is put in a magnetic field. Because each orbit contains a 'spin-up' and 'spin-down' electron, these electrons are more or less susceptible to ionisation when a magnetic field is present. This causes spectral lines to split slightly. The Zeeman-splitting constant is proportional to the bohr magneton, the minor effects are carried across by numbers. The Zeeman effect was the first indication that something more complex than what is explained in Bohr's atom.

The Landé g-factor is given as a variance from the classical magneton or gyromagnetic ratio,

		Somerfeld	Kennelley	
12.	μ_B	$= evr/2\kappa$	$= evr/2\eta c$	Bohr Magneton
		$= N\,eh/4\pi m\kappa$	$= N\,e\hbar/2m\eta\kappa$	
13.	'm_m'	$= e\hbar/2\mu$		"Magnetic Mass"
14.	k_z	$= e/4\pi mc\kappa$	$= \mu_B/hc$	Zeeman splitting

15.	f	$= \mathbf{B}`\bar{\gamma}'$	$= 2\pi\omega$	Gyromagnetic Ratio.
	$\bar{\gamma}$	$= \mu/\hbar$		
16.	g_e	$= 2\bar{\gamma}m/e$	$= 2\mu m/e\hbar$	Land e G-factor

The kinetic energy of the orbit is equal to what is needed to free it, so

17.	E	$= \frac{1}{2}mv^2$	
		$= Z^2/N^2\ mc^2/2\mathrm{k}^2$	
18.	E/e	$= Z^2/N^2\ mc^2/2\mathrm{k}^2 e$	Ionisation of Bohr atom

Rydberg's constant unifies the sparse lines of the Hydrogen atom, and similar single-electron ions. The original series are given as follows. Rydberg discovered the general series to merge all these, based on wave numbers. A and B are integers.

Balmer Series	1885	A=2	visible
Pashen Series	1908	A=3	infrared
Lyman Series	1915	A=1	ultravoilet
Brackett Series	1922	A=4	infrared
Pfund Series	1924	A=5	infrared
Humphres Series		A=6	infrared

19. Rydberg constant $1/\lambda = R_x Z^2(1/A^2 - 1/B^2)$

In the Bohr model, the Rydberg constant is explained by a transition between levels of the atom, the rydberg constant comes from the Planck relation.

	hc/λ	$= (E(A) - E(B))$		$E(A) = Z^2/A^2\ mc^2/2\mathrm{k}^2$
	R_x	$= E/hc$	$= m'c/2\mathrm{k}^2 h$	m' $= Mm/(M+m)$ (m orbiting M)
20.	R_∞	$= mc^2/2\mathrm{k}^2 h$		Rydberg Constant (infinite mass)
	$1/\lambda$	$= (1/A^2 - 1/B^2)\, Z^2(M/(M+1))R_\infty$		Wavenumber from M, Z, A, B

Here, m is the reduced mass, being $m' = Mm/(M+m) = m - m^2/(M+m)$. When M is infinite then the reduced mass becomes $m' = m$. But it was that it correctly varies that convinced Rutherford that Bohr was indeed on a correct model. Rydberg's constant is now evaluated for an infinite charge, the finite case is found by the relation $R_x = R_{\mathrm{inf}}(M/(M+1))$, where M is the mass of the nucleus in electron masses.

Morsley found in 1913, that the frequency of the farthermost lines of various atoms. The resulting lines showed that Z in Bohr's atom increased in line with the position in Mendeleev's periodic table: that is, the order in Mendelev's table comes with the central charge in Bohr's model.

The Photoelectric effect is the first indication of quantised light. Specifically, light will not ionise an electron until its wavelength is long enough. This wavelength is $f = E/h$. This gives rise to the particle nature of light.

De Broglie suggested that particles behave as waves too.

21.	λ	$= h/p$	$= h/mv$	Wavelength
22.	f	$= E/h$		frequency
	v	$= \lambda f$		velocity

In the Bohr atom, the electron in its orbit is a standing wave. The idea of standing matter-waves in potential wells is the idea behind Schrödinger's model.

9.	λ	$= (N/Z)\mathtt{k}j/mc$	de Broglie wave
10.	$2\pi r$	$= (N^2/Z)\mathtt{k}h/mc$	Orbit Length
23.		$= N\lambda$	In de Broglie waves

The Crompton effect is the scattering of light at near the speed of light, that v in eqn 21. is the same as light. The speed is reduced by the cosine of the deflection angle.

24.	λ	$= h/mc$	Crompton wave length

The model of the classical electron supposes that its its energy of mass comes from the electrostatic energy against capacitance. The ratio of the classical electron radius to the bohr orbit is $\mathtt{k}^2$.

	E	$= mc^2$	$= e^2c/\gamma\eta\kappa\, r$
	r	$= e^2c/4\pi\ \beta\eta\kappa\ mc$	
25.	r_e	$= h/2\pi \mathtt{k} mc$	Classical Electron Radius

The measure of thompson scattering derives from the classical radius. The scattering cross section is defined as I_s/I_o. where I_s is measured in solid angle (solid radians), and I_o in area. The unit is in area per solid angle. In thompson scattering, it is the slow-moving electrons that do the scattering.

26.	s	$= \frac{8}{3}\pi r^2$	Thompson Cross Section

Schrödinger takes de Broglie's wave model, and uses it to solve for standing waves of electrons in the Bohr model. In the equation below, V represents the kinetic energy, and Ψ the wave function being sought. The constant $\hbar^2/2m$ is Schrödinger's constant.

27. $$i\hbar\tau\Psi(r,t) = \left[\frac{-\hbar^2}{2m}\nabla^2 + V(r,t)\right]\Psi(r,t)$$

6 Gravity

Newton's gravity can be modeled on a radiant field. The following equations show the inverse square law, and the derived modern form. In this form it serves well enough to place objects in orbit and on the moon and Mars. Although Newton's G is known imperfectly, the gravitational tractive GM is quite well known.

Classical Form	$\mathbf{F} = \text{‘}G\text{’}mM/\mathbf{r}^2$	
Field	$\mathbf{F} = m\text{‘}\mathbf{g}\text{’}$	
Potential	$g = \nabla\text{‘}\Phi\text{’}$	
New style	$\nabla \times g = 0$	$\nabla \cdot \mathbf{g} = -4\pi G\rho$
G-tractive	$\text{‘}GM\text{’} = GM$	
gravitoelctric	$M = \text{‘þ’}Q$	$\mathbf{F} = GMM/R^2 = QQ/\gamma\epsilon R^2$

Gravity at the scale of the solar system is essentially non-relativistic, to the end that over hundreds of years, no one has saw fit to impose a model of motion different to what is felt on the earth. Moreover, even the fastest of objects in the solar system - Mercury - the relativistic effect is so slight that it is listed as an anomaly.

Although for practical purposes, one can treat gravity in terms of the newtonian model, neither of these are relative to a constant speed of light. One must add additional elements of the order of $1/c^2$ to preserve the relativity. Such is so slight that even the more sensitive meters are only just able to select this measure.

6.1 Co-Gravitation

As electromagnetism was prototyped on gravity, it comes about that gravity is reformulated on electromagnetism. Oliver Heaviside first demonstrated that, where the field of gravity moves at a finite speed, then it behaves akin to the electric field in electromagnetism.

Einstein's general relativity gives formulae similar to Heaviside's, but include extra terms due to relativistic elements. In this theory it is normal to deal with four-vectors, which we do not propose to do here. None the same, some of the terms have an additional factor of 2 or 4, which is not shown here.

Table 2.1: Gravitational EM Units

	$þ^{-2}$	$þ^{-1}$	$þ^{0}$	$þ^{1}$	$þ^{2}$
β^-1	K_e G				
β^0	μ	$\mathbf{D}$ $\mathbf{H}$		$\mathbf{E}$ $\mathbf{g}$ $\mathbf{B}$ $\mathbf{K}$ $V\Phi$	ϵ η
β^1		$\mathbf{P}$ $\mathbf{M}$ Q M		$\mathbf{K}$ $\mathbf{J}$ $\mathbf{j}$ $\mathbf{d}$	
	Ω, H	A, C	W, J	V, Wb,	S, H

The constant 'þ' = $\sqrt{K_e/G}$ is the ratio between electrostatic charge and gravitational mass, such that if the mass is divided by þ, the same force arises from electrostatic charge.

When $M = Qþ$, one can then find $g = e = E/þ$, and various

$\nabla \cdot \mathbf{D} = \rho Q/\beta$	$\nabla \cdot \mathbf{d} = \rho M/\beta$	$\mathbf{d} = M/\gamma \mathbf{r}^2 = þ^2\mathbf{b}$
$\nabla \cdot \mathbf{B} = 0$	$\nabla \cdot \mathbf{b} = 0$	$\mathbf{b} = \mathbf{g}/c = \mathbf{K}$
$\kappa\nabla \times \mathbf{E} + \tau\mathbf{B} = 0$	$\nabla \times \mathbf{e} + \tau\mathbf{b} = 0$	$\mathbf{e} = \mathbf{g} = \mathbf{F}/M$
$\kappa\nabla \times \mathbf{H} - \tau\mathbf{D} = \mathbf{J}/\beta$	$\nabla \times \mathbf{h} - \tau\mathbf{d} = \mathbf{j}/\beta$	$\mathbf{h} = c\mathbf{d} = þ^2\mathbf{e}$

Because the model is surprisingly good, although not perfect, one might suppose that c is promoted from an electromagnetic quantity, to a property of space, and that the carriers of electric field and of the gravity field, are massless particles that ride on the space-time conversion factor.

One can see that the relation between Electric and gravity is $Qþ$, $\mathbf{D}þ$, $\mathbf{H}þ$, $\mathbf{E}/þ$, $\mathbf{B}/þ$, $1/\gamma G = \epsilon þ^2$, $\mu/þ^2$.

In any case, one might be tempted to follow the fpsc practice of setting.

Inverse square law	$\mathbf{F} = cQq/4\pi\mathbf{r}^2$	$\mathbf{F} = cMm/4\pi þ^2\mathbf{r}^2$

6.2 Quantum Gravitation

The units of Stoney and of Planck, are more simply written in terms of the values given above. Stoney's system was implemented before the quantum constant $\hbar$ was discovered. The units represent the scale of quantum gravity.

	Length	Weight	Time	Charge
Stoney	$e/\gamma þ c$	$eþ$	$e/\gamma þ c$	e
Planck	$q/\gamma þ c$	$qþ$	$q/\gamma þ c$	$q = e/\sqrt{\alpha} = \sqrt{\gamma\hbar}$

When rationalising these systems, one should replace γ with β, but keep þ, c and the charge unit. The effect of doing this causes the length and time unit to increase by a factor of 4π. But one notes that the units of $\hbar$ is correctly $\mathrm{L^2M/T\Theta}$, where Θ is a measure of angle. So the length changes from a radian-length to a double-wavelength.

Chapter 3

Tables

7 A Story of Units

The proto-electric and proto-magnetic systems use the newtonian inverse-square-law, but because these systems are defining a new constant, it is possible to set these to unity. That is, the proto-electric equations have $\gamma = \epsilon = 1$ and the proto-magnetic equations set $\gamma = \mu = 1$.

When one uses proto-electric units for electrical quantities, and proto-magnetic units for magnetic quantities, one is just using the ordinary mechanical system, eg c.g.s. or f.p.s.. It's only when you implement the conversion factor κ, and start having electrical units for magnetic quantities, and so forth, that one starts to talk of 'electrostatic' and 'electromagnetic' units, or e.s.u. and e.m.u. The units are then written, eg c.g.s.e. or f.p.s.m.

The use of a mix of e.s.u. and e.m.u. make a system that is now-days called the gaussian units, although this is an honour-name, since Gauss was long dead by the time Maxwell wrote his book on electromagnetism. Maxwell himself gives the dimensional analysis of these units.

The underlying relation is that the gaussian preserves the pre-existing proto-units, and allows an indirect scale, by way of setting $\kappa = 1$. The relation to be observed is $\epsilon\mu c^2 = \kappa^2$, of which any two might be set to unity, and the third one evaluated.

The e.s.u. sets $\epsilon = \kappa = 1$, the e.m.u. sets $\mu = \kappa = 1$, and the gaussian sets $\epsilon = \mu = 1$. A fourth solution, explored severally by Fitzgerald, by Kennerley and by myself, gives $\epsilon c = \mu c = 1$, which eliminates the separate use of electric and magnetic units. The use of this fourth scale did not take on because its units do not match the pre-existing e.s.u. and e.m.u scales.

7.1 Practical Units

The units defined by early experimentalists, was to define resistances in terms of lengths of wire, and voltages in terms of voltaic cells. This lead to units like a resistance-unit equal to ten yards of number 15 wire, or the Latimer-Clarke cell.

Agreement was made that while the previous practice would continue, one would quote the measure in the decade e.m.u. nearest a pair of units: the Daniell's cell, and the Siemens resistance. A good deal of names were discussed before a list was settled on. An Ohm is then 1.063 Siemens units, a Volt is near a daniels cell. One derives an ampere as a volt per ohm, a coulomb as an ampere-hour, a farad as coulomb per volt, a watt as a volt by an ampere, and a joule as a watt-second. The unit called henry, was originally a quadrant, since it represents $1 \cdot 10^7$ metres or a quadrant. Later it was called a secohm and then a henry.

The 'international' part comes from agreement that nations would replace their own legal implementations of the volt and ohm, by ones of common agreement. For a while, the ampere, defined in terms of the silver faraday, had some currency. By 1947, one could implement the e.m.u. definition directly at this scale, and the international practical definitions were depreciated.

Because the volt-ohm-second do not implement a full system of units, one could pretty much use it with whatever local units. 'Volts per inch', for example, is not uncommon, and is not so much a mix of SI and fps units, but a free-standing electric system and the local measures.

Of greater interest is the set of units called c.g.s. practical, or 'Hansen' units. The practical units were used of the electrical units, and the magnetic units were the straight unmodified c.g.s. e.m.u. In 1930, the IEEE were coerced into approving names for four of these: gauss, maxwell, oersted, and gilbert.

There was then a proposal to name the e.s.u. by the prefix 'stat-', and the e.m.u. by the name 'ab-'. While widely used, it was never sanctioned by an official body. However, with rationalisation, one should either avoid these names, or also use additional suffixes to deal with the 4π factor.

7.2 Rationalisation

Rationalisation is the term used for converting from a $\gamma = 1$ to $\beta = 1$. Such a move is logical when one starts electrical theory from something which resembles Maxwell's equations, as Oliver Heaviside did. However, one can see that if $E^2\beta\eta\kappa$ (ie units of weight and force), is to be preserved, then there are many different ways of installing the 4π into different values.

The method followed by Heaviside is to keep ϵ and μ unchanged, which makes the size of the charge unit larger by a factor $\sqrt{4\pi}$ and the unit of potential smaller by the same factor. Heaviside suggested reforming the practical units, by now thought of as c.g.s. units, by similar factors, so that the value of $\mu = 10^{-9}$ H/cm. He then discusses in the same reform, one could equalise the powers to the same number, eg setting the new joule to 10^9 erg.

Lorentz followed Heaviside, both in using units that differed by a factor $\sqrt{4\pi}$, and by using Maxwell's equations as a starting point. But he uses a symmetric system similar to Maxwell's system, complete with $\epsilon = \mu = 1$.

One could, as with the response from the British Association and others, keep the bulk of the e.m.u. and the various quantities, if charge, coulomb's constant, and ampere's constant, were kept identical, and the size of $\epsilon = 1/4\pi$ and $\mu = 4\pi$.

One can suppose a third rule, which sets $\epsilon = 4\pi$ and $\mu = 1/4\pi$, which would preserve the unit pole. The unrationalised systems are then a *mix* of the three rules describe above.

8 The FPSC and the UES

Leo Young[You69] gives quite an extensive account of efforts to reconcile the diverse formulae that arise from the older theoretical physics and that which is now taught in schools. While this is generally done by adding constants to the individual author's favourite system, Young's solution is to apply new dimensions to these. Such constants are to be read as system constants, but can be given some sort of physical meaning.

The basic approach is to take a body of equations in two different systems, such as c.g.s. and SI, and compare the formulae. Where there is a difference, one might introduce a constant, say $S = U = 1$ in SI, and then put $S = 4\pi$ and $U = 1/c$ in the c.g.s.

$c\nabla \times \mathbf{E} = -\tau B$	$\nabla \times \mathbf{E} = -\tau B$	$\nabla \times \mathbf{E} = -U\tau B$
$c\nabla \times \mathbf{H} = 4\pi j + \tau D$	$\nabla \times \mathbf{H} = j + \tau D$	$\nabla \times \mathbf{H} = SUj + U\tau D$
$\nabla \cdot \mathbf{D} = 4\pi\rho$	$\nabla \cdot \mathbf{D} = \rho$	$\nabla \cdot \mathbf{D} = S\rho$
$\nabla \cdot \mathbf{B} = 0$	$\nabla \cdot \mathbf{B} = 0$	$\nabla \cdot \mathbf{B} = 0$
$\mathbf{S} = c(\mathbf{E} \times \mathbf{H})/4\pi$	$\mathbf{S} = \mathbf{E} \times \mathbf{H}$	$\mathbf{S} = (\mathbf{E} \times \mathbf{H})/SU$
$\mathbf{D} = \epsilon_r\mathbf{E} = \mathbf{E} + 4\pi\mathbf{P}$	$\mathbf{D} = \epsilon\mathbf{E} = \mathbf{E} + \mathbf{P}$	$\mathbf{D} = \epsilon\mathbf{E} = \mathbf{E} + S\mathbf{P}$

The equation $\Phi = SQ$ might suggest that S could be associated with solid angle, and that the c.g.s. unit of flux is represented by Q.steradian, while the rationalised form might be represented by Q.sphere. One then reads the unit as charge times solid angle. Likewise, magnetic flux has the dimensions of VUt, is supposed to arise from magnetic charge, which is variously VUt/radians or VUt/spheres.

Likewise, the equation $I = QU/t$ might be read that the Ampere-current is measured in ampere-turns, and that such arise from Q/t amperes by U turns.

Young spent a whole chapter dealing with this notion, before suggesting it was a bad idea, because not all solid angles and turns are being modified: only particular ones.

8.1 Prefix Rules

Before rationalisation became quite common, there was a suggested practice, of writing the c.g.s. electrostatic units as the practical unit, prefixed by *stat*, and the corresponding electromagnetic unit prefixed by *ab*. So, the erstwhile unnamed unit of c.g.s.e. charge is a *statcoulomb* βC and the c.g.s.m. unit becomes an *abcoulomb* αC.

When rationalisation is applied, the coulomb of charge produces a coulomb of flux, whilst a statcoulomb of charge produces 4π statcoulombs of flux. One can no longer apply the common conversion factors.

None the same, the idea that the factor of c can be handled by a named rule forms the basis of the U.E.S. rules.

8.2 The Rationalisation Rules

	Prefix (c)			Suffix (4π)	
G	Gaussian	0	U	Unrational	0
J	Indirect	$2\varpi - E$			
E	Electric	$\varpi - \eta$	I	Conventional	$\beta - \eta$
M	Magnetic	$\varpi + \eta - E$	Y	Trittade	$\beta + \eta - E$
W	Derational	$\varpi + \beta - E$			
V	Base	ϖ	L	Light	β
D	Electrodynamic	$E - L$	O	Heavy	$\beta + E - L$
N	Symmetric	$\varpi - \frac{1}{2}E$	R	Rational	$\beta - \frac{1}{2}E$
C	Sym. ED.	$\frac{1}{2}E - L$			

9 UES and Systems

9.1 The Units in various Systems

		e.s.u.	e.m.u.	SI	BR=fpsc
E	E	**dyn/Fr**	Mx/cm s	V/m	*Gv/ft*
H	$E\eta$	dyn/Sa.s	**Oe, Gb/m**	A/m	Gv/ft
D	$E\eta\varpi$	**dyn/Sa.cm**	Gb.s/cm^2	C/m^2	By/ft^2
B	$E\varpi$	erg/Fr.s	**Gs, Mx/cm^2**	T, Wb/m^2	*By/ft^2*
P	$E\beta\eta\varpi$	**Fr/cm^2**	Bi.s/cm^2	C/m^2	*Vb/ft^2*
J	$E\beta\varpi$	Sa.s/cm^2	**Up/cm^2**	Wb/m^2	Vb/ft^2
K	$E\beta$	**Sa/cm**	Up/cm.s	V/m	Or/ft
M	$E\beta\eta$	Fr/cm.s	**Bi/cm**	A/m	*Or/ft*
w	$E^2\beta\eta\varpi$	dyn/cm^2	=erg/cm^3	N/m^2	pdl/ft^2

c.g.s. Gaussian GU units in **bold**, Indirect JU units in regular face.
fpsc Dimensions agree between SI and FPS when the fps is in *italic*.

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