PolyGloss v0.05

This is my multidimensional glossary. This is a cut on the terms as viewed by someone who can visualise four and higher dimensions, and finds the commonly used terms to be more of a hinderance.

Normal Entry

A normal entry appears in this style. The header appears in normal bold font.

Comment

A comment appears with the header in italics, such as this entry. Comments are not suggested for use based on this glossary, but are more in line to allow the reader to find the preferred entry.

Decimal (dec)

Numbers written in groups of three, or continuous, and using a full stop, are decimal numbers. Each column is 10 times the next. If there is room for doubt, the number is prefixed with "dec", eg dec 288 = twe 248.

There are no special symbols for the teenties (V0) or elefties (E0), as these carry. twe V0 = dec 100, twe E0 = dec 110. The letter E is used for an exponent, but of 10, not 100, so 1E5 is 100 000, not 10 000 000 000, or 235.

```
ten               10       10     ten million  10 000000     594 5340
hundred          100       V0     100 million 100 000000    57V4 5340
thousand        1000      840     1 milliard 1000 000000  4 9884 5340
10 thous       10000     8340
100 thous     100000    6 E340
1 million    1000000   69 5340
```

Twelfty (twe)

Numbers written in groups of two or four, and using a colon (:) for a radix are numbers in base 120. Pairs of digits make up a single place, the first is 10 times the second: so in 140: = 1 40, the number is 1*120+40. If there is any doubt, the number will be prefixed with "twe", eg twe 248 = dec 288.

The digits for the teenties (100) and elefties (110) are V and E, so the number 10×10 = V0, 10×11 = E0, 10×12 = 100. Also H is used as an exponent, eg 1h5 = 100 0000 0000 = 1×120^5

```
                                   ten                      10           10
hundred          100          120  ten hundred            1000         1200
thousand      1 0000        14400  ten thousand        10 0000       144000
cention     100 0000    1 728000   ten cention       1000 0000   17 280000
million 1 0000 0000  207 360000    ten million 10 0000 0000   2073 60000
```

Notion and Notation

A well chosen notion leads to a notation that assists in the discovery of polytopes. We should note, that Wythoff and Stott, who discovered many of the polychora, are associated with operations and notations that effortlessly produce the mirror edge figures. The great many of these come as a result of filling in the holes.

My own exploration of the compound polychora was largely driven by getting the right notions about how the stations of the twelftychoron work. I discovered 96 in a single day, this reduces to seven sets of compounds, that accepting any one automatically accepts the remaining seven or fifteen.

The bringing together of diverse idioms can provide wonderful insight. The idea of seeing polytopes as integers leads to compounds as modulo classes. Since Integers and Modulo classes are often represented by polytopes and tilings, it is not a great difficulty to extend the notion to cover polytopes that are not representing of integers.

In folding together names like this, one must always be careful that one does not give rise to conflicting imagery. While it is useful to describe stations as modulo classes, one should always be wary that the idiom may no longer continue to be the same thing.

Many Dimensions

The terminology of the polygloss is meant to reflect what one might find in six to eight dimensions. This particular area is much richer than the that of three and four, and the different senses or meanings of words give rise to different polytopes.

To the extent that new terminology is needed, I have decided to do a "Latin and Greek" approach, of building words from base stems, to describe a great number of words with concise meanings. A large family of these are given together at the entry on "Polytope".

The existence of this process has allowed us to free up a number of words, many of which have other useful meanings worth preserving. In six dimensions, the role of three dimensions is not that different to the role of one dimensions in our space. It does not bound. A figure made with solid surchora would be as hollow and empty as the ones that Da Vinci produced of solid edges. To apply some word that suggests a solid or bounding nature is the same as to suggest that any stake driven in the ground is a fence.

	Meaning 1	Meaning 2	Polygloss alternative
cell	solid surtope	3D surtope	surchoron
face	bounding surtope	2D surtope	surhedron
sphere	solid sphere	3D sphere	glomohedron or triglome

Walls and Bridges

Part of the difference in this gloss to what one finds elsewhere is that I look at multi-dimensional terminology from the point of view of someone who would live in the dimension under discussion. Walls and Bridges should serve the same function regardless of the dimension one is in: walls divide, and bridges unite.

Where a word has a sense of dividing space, it is allowed to move with the dimension of space: so plane, surface, in, out, are all concepts that connect to the ambient space, while vertex, edge, line, are fixed things. For all of the medial dimensions, I invent new words. So a face is always the bounding elements of a polytope, regardless of dimensionality: that is, it is a polytope wall. That in 3D, a face has 2D is irrelevant to this exercise: a 2D element gets a new name *surhedron*, a "hedron" on the surface of a polyhedron.

Many of the words that change meaning are also glossed in italics, giving the reasons for the change.

Tiling vs Polytope

The form of geometry I use makes no distinction between the surface metrics of a polytope or a tiling. A tiling of hexagons can undergo the same transformations as a dodecahedron or cube. But it is convenient to talk of a tiling as being in the dimension it fills, and without a bound interior, while a polytope potentially bounds an interior, and does not cover all space. So a dodecahedron is a 3D polytope or polyhedron, while a tiling of hexagons is a 2D tiling or apeirohedron.

All the same, figures inscribed in covering Euclidean or Hyperbolic space are described as apeirotopes, rather than polytopes. Note however that the term infinitope is used to describe the same figures when they act like polytopes (ie with an interior, etc). The cells of an {8,3,4} are right- angled {8,3}, which would normally be described as a 2D apeirohedron, rather than a 3D polyhedron or infinitihedron.

A figure might be described as an E4 tiling or H5 polytope. This means that its surface can be treated as a tiling that covers a 4D euclidean all-space, or the thing can be regarded with an interior, as covering part of H5 space.

Curvature of Space

No distinction is made to the nature of space that the figure is in for the naming, although it is described as Elliptic, Parabolic or Euclidean, and Hyperbolic.

- **Elliptic** refers to figures inscribed on a sphere. Although the term commonly is used for the spherical space identifying opposites, the other geometries do not have such descriptors, and for this, mono- is invoked.
- **Parabolic** refers to figures inscribed in a plane of zero curvature. This is the limiting space as the diameter of the sphere becomes increasingly large, it is also the general geometry of real things that we experience. **Euclidean** is a synonym of parabolic.
- **Hyperbolic** refers to figures inscribed in a surface of negative curvature. While we might not be able to experience this directly, the geometry is a perfectly valid substitute for the Euclidean.
- **Mono-**, when prefixed to a name, means that opposite points of the geometry are identified. Lines in such a geometry cross in only one point, rather than the usual two. The second crossing in Parabolic or Hyperbolic spaces are not in the realm of accessible points.

Note, however, that we might describe a figure as an E4 or H5 tiling. The letters S, H and E stand for spheric, euclidean, and hyperbolic, and the number refers to the dimension of all space. A tiling covers all-space, while a polytope covers a fraction of all-space. A {3,6} may be described as a E2 tiling, or an E3 polytope, even though the latter could also be a H3 polytope. If the {3,6} is particularly desired to be in hyperbolic space, it might be described as "an E3 polytope in H3", or "polytope in H3".

Circle Drawing

This is a presentation of geometry through a series of Euclidean circles. Distances are a function of two points and the curvature of the ruler. In the case of circle-drawing, a polytope's distances remain unchanged except for a scalar, as it is allowed to grow and contract: the figure may vary from flat to having acute angles.

The prototype is drawing circles on the surface of a sphere. These points have the geometry of both the sphere and the Euclidean space the plane is in. Because a large amount of the work in this gloss mainly concerns drawing shapes inside spheres, the assumptions of circle-drawing are most appropriate here.

Just remember the distances are not the same as in the space: for the sphere, they're internal chords, shorter than the surface arc. In the hyperbolic space, they follow a horocycle, and are asymptotic on the exponent of the straight-line length. None the less, the same crooked ruler gives the same measurements, and there are functions that convert the crooked ruler into a straight one.

Version Notes

- v0.05 Added new entries for semiates, horo- and pseudo-, and tided up entries, like hole.
- v0.04 The first 4DOS version, using a markup language based on 4HELP, and HTMLised by REXX. Lots of new entries added. Corrections based on emails from v0.01 and v0.02 incorporated.
- v0.03 Last of the Amipro versions. Some adjustments were made after the freeze, but work began on the current 4DOS version. Not released.

The Glossary

Above-Below tiling

This is a tiling formed by cutting space by planes. Each plane has a top and bottom side. There are cells completely above every face, and cells completely below every face. A journey from a complete-top cell to a complete-bottom cell takes N steps.

The vertex figure is a simplex-antiprism. Removing any diagonal gives a series of vertices lying in the same plane, and hence, the lesser-dimensional member of the species. Labeling the top cell as A, and the bottom as B, all cells have vertices lieing in the top and bottom, a cell with 3 A vertices and 2 B vertices would be AAABB.

An example of the above-below scheme is the cubic lattice. A complete-top cell might be represented by all odd coordinates: eg OOOO. A complete-even cell would be a complete-bottom cell EEEE. Crossing any plane converts an O to E if going down, and E to O if going "up". Note that the O and E can represent different lengths, this turns OEEE, OOEE and OOOE into generalised prism-products of lesser-dimensioned cubes.

The importance of AB tilings is that they contain the previous example in a plane. For example, if we drop any given letter of the above example, we get what happens in a plane perpendicular to that axis: thus EE.E becomes EEE.

AB tilings occur in the form of the runcinated {;p,3..3,p;}, and the loop truncate {;p,3..3,p,q:}. The typical AB cell is found by taking a nodes to the left, and b nodes to the right of 3..3,p;q;p,3..3. In the case of the runcinate, q=2. For example, an AAABB cell of {;3,3,3,3;4:} can be found as a3a3a4b3b, or {3,3;4;3}.

Because an AB tiling contains a lesser AB tiling of the same species as a cross section, we see that {p,3,3,p} contains {p,3,p} contains {p,p} for all p, and {p,3,3,p,q:} contains {p,3,p,q:} contains {p,p,q:}. This means that eg, {5/2,3,3,5/2} would contain {5/2,3,5/2} and {5/2,5/2}, but the last is clearly fractionally infinite, so therefore is the lot.

Examples of AB tilings are as follows. The number is p,q,p.

- 323 The only finite polytope, a runcinated simplex.
- 424 The cubic, with even and odd cells.
- 333 The t-truncate, a tiling of a simplex and its polytruncates.
- 525 The series {;5,5;}, {;5,3,5;} and {;5,3,3,5;}
- 343 The series {;3,3;4:}, {;3,3,3;4:}, {;3,3,3,3;4:}, {;3,3,3,3,3;4:}
- 434 353 and 535 are only in apeirohedra and apeirochora.

All-Space

A space where a given situation is solid, that is, without surplus dimensions. One can treat all-space as less than everything. For example, the surface of the Earth represents an all-space in the form of a 2D-sphere, even though it is immersed in a higher 3D space.

Altitude

Those dimensions of a simplex or exotic prism not present in the base. While we speak of height of a pyramid, in some exotic prisms, the whole thing can be altitude. The bases fall in the transverse symmetry.

#Ambiate

Conway's notation of rectate or rectify. The # refers to the dimension of the surtope that becomes the vertex.

#-Angle

This is the measure of all-space around a point, where # refers to the dimension of all-space. In 3D, these are called plane and solid angle.

Angle measure

The measure of the portion of all space occupied by a figure at a point. This is measured in the space where the polytope in question is solid, and is either measured in area or fraction of space.

In units involving the radian, the angle is presented as a sphere surface, and this is then measured in terms of the zero-curvature radius, in surface prism-radians or surface tegum-radians.

For twelfty-measure, the practice is to treat the angle as a fraction of all-space, the normal unit being 120^-x, where 2x is the largest even number not greater than the dimension.

Also in use is degree like units, the degree, grad. A circle is rated as C degrees, and the sphere as 2C degrees excess, or C^2/pi square degrees. C = 360 for degrees, and 400 for radians.

Anglutope

A "corner" polytope. For example, the single vertex of a dodecahedron is the corners of three different pentagons. An edge of a dodecahedron is the sills (corner-edges) of two pentagons. An angluhedron is a 2-corner.

Antiprism

An exotic prism formed by a figure and its dual. It shares many interesting features with the polyprisms, which are antitegums.

Antiprismal sequence

The process of expansion or runcinating.

Archimedean figures

This is a set of convex uniform figures not including the prisms or the platonic figures. In practice, the set usually enumerated is the convex uniform figures, the archimedean ones being the not-elsewhere-included set. Most Archimedean figures are found by mirror-edge construction, or one of the snub rules.

Antitegmal cluster

The result of replacing each face of a polytope with an antitegum of that face, meeting at the centre. This is the result of a strombiation.

Antitegmal sequence

The process of vertex-beveling, giving the truncates and rectates. The antitegum is the intersection of pyramids of a figure and its dual, sharing a common altitude, and the base of each pyramid is at the apex of the other. The effect is to give the faces of the dual descending on the first, and the common intersection being the truncates and rectates.

Antitegum

An exotic tegum formed by a figure and its dual. This is the common intersection of a pyramid of a figure, and the pyramid of the dual, where the base of one is at the apex of the other.

Apeirotope

Faces without end. This implies a honeycomb. For example, the apeirochoron carries 4D names, but is a tiling in 3D. When the apeirotope is furnished with a real interior, it becomes a polytope. A polytope with infinite number of faces may be called an infinitope.

```
apeirogon      1d tiling     apeiropeton   5d tiling
apeirohedron   2d tiling     apeiroexon    6D tiling
apeirochoron   3d tiling     apeirozetton  7D tiling
apeiroteron    4d tiling     apeiroyotton  8D tiling
```

```

     (no space)              (empty space)          (empty space)
                          . . . . . . . . . . .    . . . . . . . . . .
  ---o-----o-----o----    ---o-----o-----o-----    ---o-----o-----o--
                          +++++++++++++++++++++    ++/+\+++/+\+++/+\+
     (no space)           +++(solid interior)++    +/+++\+/+++\+/+++\
                          +++++++++++++++++++++    o-----o-----o-----
                          +++++++++++++++++++++    . . . . . . . . . .
    apeirotope               infinitope              laminatope
```

#Apiculate

The dual of the #truncates, formed by the convex hull of a figure and its dual of varying sizes. Apiculates form when none of the dual-elements are concentric. The faces are pyramids of the #margin and the #-1 edge of the dual. When the dual-elements do match, a #surtegum forms.

Archimedean

This term is used to describe uniform polytopes that are neither platonic, or of the prismatic classes. While the term in itself refers to the polyhedra, it generalises quite well into any kind of geometry and dimension. So one might describe archimedean horochora (E3 tilings).

The dual of archimedean is the catalan.

area

The solid measure of a plane. In the context of this glossary, a plane is an N-1 flat, rather than a 2-flat.

#-Army

[Olshevsky] A grouping of units that share the same surtopes as far as # dimensions. Units of the same surtope army share those surtopes, and all lesser ones: so members of the same regiment share the same vertices and edges.

```
                  Name       Leader                      faceting
 --------------------------------------+-------------------------
  vertex-army      army       general  |  margin-army
  edge-army        regiment   colonel  | 2margin-army
  surhedron-army   company    captain  |
```

#-Army leader

[Olshevsky] The notional polytope that other units share the same #-army share. The #-Leader is 0~ =General, 1~ =Colonel, 2~ =Captian.

#-Army set

[Klitzing, Krieger] An extension advanced where the shared elements are either a super-set or sub-set of the Army. As the icosahedron's vertices and edges are a superset of those of the pentagonal antiprism, the icosahedron regiment is a super-regiment of the pentagonal antiprism.

Around

The sense here is in the orthospace: as one may dance around the maypole.

Askew

The dual of a skew polygon, having rotary symmetries at the vertex. Such a polygon occurs as the girthing polygon of a {p;q}, with rotation at the vertices and reflections at the edges. These may be written as {p%}. Askew polygons occur as the faces of the Petrie-Coxeter polyhedra.

Askew Chords

A chord across the polygon where the out-vector changes: that is, the same side of the face changes from exterior to interior. This is usually as a result of a crossing surtope. Surtopes do not mark the location of askew chords: that is, the chord is in the interior of several surtopes, not the boundary of these surtopes.

Attol

A 6-face or N-6 edge.

Base

Any of the figures involved in a tegum, prism, pyramid or comb product. The term is used for any of the apex-figures of an exotic simplex, or the

Between

Where the sense is not elsewhere defined, this is to be taken as lying in a sphere diametric on the two points, and solid in "all-space".

Bevel

A Conway operation that produces the omnitruncate directly from the regular figure. This places a vertex in the centre of every flag.

Bevel, surtope

The removal of parts of a polytope by the decent of planes. These come in the direction of the named surhedra: vertex-bevel, edge-bevel &c.

Bi-

A prefix meaning 'two' or 'twice'. If the interrelation between the two is implied, then one should use "di-".

Binary

A class of polytope with intersecting surfaces, where the density of the surface alternates on each crossing. For normal polytopes, this is a modulus 2 of the actual density, but binary operation also allows a non-orientable surface to contain a proper interior.

Askew chords are used a lot in binary polytopes.

Bipyramid

A tegum. The sense here is an exotic tower made by joining two pyramids at their base. In practice, the figure is actually realised as an exotic tegum, where the two bases are identical.

Blade

A name being considered for the second-series face.

Blend

A figure derived by removing common features from a positioning of polytopes. In practice, one joins a number of separate polytopes, including noids to produce some sort of compound. One can then create a second assembly of polytopes that agree in surtopes in every manner. One can describe any of the non-noid polytopes as a blend of the remaining ones.

Borromeachoron

An infinite family of uniform hyperbolic apeirochora, shown in Charlie Gunn's video "Not Knot". The vertex figure consists of a snub tetratetrahedron, the vertex figure having 12 {p}{} and 8 {4,3}. When p=4, this becomes the {4,3,5}

Borromeateron

The 4D reflex of the Borromeachora, the vertex figure is a s{;3;4,3}, with appiculated icosahedra. It forms for p=3,4 and 5, but only for p=4 are all sides equal. This is the {4,3,3,5}.

boundary

Something that bounds. The implication here is that it should separate the interior of something, in the space that it is inscribed in. A second boundary would bound (some part of) the first boundary.

#-Boundary

D H Y Sommerville *The Geometry of N dimensions* uses it to refer to what I glossed as an #-edge.

Bower's Acronyms

A series of short names proposed by Jonathan Bowers, and having some currency. These take Bower's extensions to the existing names, and produce short names, based on "significant" letters in the source name. When one frequently uses a range of uniform figures, these names become a blessing.

Bower's naming

Jonathan Bowers proposed this series to name the assorted regular truncates. They are based on the various kepler names, and is extendable to higher dimensions, by the addition of additional prefixes.

```
  (number)  cello-  prismato-  rhombi-  trunc- ated

   5432xtrpc..
     ooxoxx      tri prismato rhombi          ated
      oxoox      bi  prismato
       xxxo                rhombi trunc     ated
```

Small and great are also used, small means that all the values between the first and last named nodes are unmarked, and great means that they are marked.

Branch

The second-series name for an edge.

#Cantellated

The result of rectifying a #rectate. The faces are cantellated, and news prism-face appears. The dual is the apiculated #surtegmal figure.

#Cantitruncate

The result of truncating a #rectate. The faces become cantitruncated, and new prism-faces appear.

Captain

The 2-army leader, which other company members share the same vertices, edges, and surhedra with.

Cartesian Product

A product that combines two polytopes, etc, such that it contains all points, that lie in two orthogonal spaces, one passing through one base and one in the other. If the orthogonal spaces cross the bases to include points in both bases, it is also in the product.

This corresponds to the union of the prism and comb products.

Catalan

A class of uniform-face polytopes, neither regular or tegmal. The vast majority of these are wythoff-mirror-margin. While the term starts off with catalan polyhedra, it can equally describe any kind of polytope, such as catalan pseudoterons (H4 tilings)

Cavity

A space on the interior of a figure, not connected to the surface. If this and the surface are counted as a unified polytope, then the cavity must be treated as an N-dimensional hole.

Cell

A solid polytope that is part of a honeycomb. [The sense here is as a bubble in a foam. Such have the same dimension as all-space]

cell

A surchoron. Such appear as bubbles if the surface is treated as a 3D spherical honeycomb.

Chord

A line of zero-curvature joining two points. In hyperbolic space, this line is longer than the straight line. A space woven by chords has Euclidean geometry. In hyperbolic geometry, a chord is longer than that the flat length. This term derives from the shortest distance through the interior of a curve. The sense here is that this shortest distance is always measured on a Euclidean, rather than flat, space.

#choron

(G. Olshevsky) A prefix meaning 3-edge. When used with a plural prefix, this suggests a 4-dimensional polytope. This is Johnson's reduction of the original #chameron.

Circle

A line in a 2-space, equidistant from a fixed point. As with all spheres, this may include the interior as well. (2) A sphere of lesser dimensions than a second sphere. (3) An angle representing all-space in 2 Dimensions.

Circum

surround, in the sense of "all space".

Colonel

A 1-army leader The notional polytope that other units of the same 1-army (or regiment) share vertices and edges with.

Comb product

This is the product that gives rise to the tiling of measure polytopes in space.

The comb-product acts such that neither the nulloid or interior are part of the product, the net result is to reduce the dimension of the product. In practice, this is a 'cartesian product of surfaces'.

On polytopes, this leads to a tiling of a spheric-prism space by prism polytopes. The most common is the squares or cubes of a bi-polygon or tri-polygon prism, but it applies to any shape.

Of Euclidean tilings, this leads to the cartesian product of the bases. A {3,6} is in effect, the surface of a 3D polytope, and the product is in effect, the surface of a 5D polytope, or a 4D tiling.

In Hyperbolic, horotopes are real polytopes, not reducible to tilings, and the product of horohedra (3D figures) yields a horoteron (5D figure).

Company

A 2-army. Such units share common vertices, edges and surhedra. The leader is a captian.

Complex polytope

A polytope that arises when dyadic symmetries are allowed to exceed two: for example, a line might have three vertices. Coxeter discussed these at length in Regular Complex Polytopes, and showed their relation to complex numbers.

Compound

A group of separate polytopes being treated as one. Such might allow symmetry actions that bring one polytope to another, or separate different polytopes set in the same symmetry. Compounds occur among the stellations and facetations of many polytopes.

A compound is a blend of separate polytopes, where the interior is shared.

#-Content

The measure of n-space, where it is solid.

#-Content measure

Any unit that is used to measure the extent of content. For example, the polyprismic, polytegmal, or polyglomic units are the content of the unit-edged measure polytope, unit diameter cross-polytope, and unit diameter #sphere.

Conway-Kepler rule

The multidimensional reflex of Conway and Kepler's remarks that the expand and bevel operators are the ambiate and truncate of the ambiate. In 4 and higher dimensions, this gives:

```
                #ambiate     =    #rectate
    ambiated    #ambiate     =    #cantellate
    truncated   #ambiate     =    #cantetruncate
```

Conway operators

A series of transformations that produce the uniform and Catalan figures directly from the regular figures. These are described as local surface operations (eg place triangles here). George Hart added another two to this, as well as writing an applet for rendering them. In this list, the first operator produces an archimedean figure direct, the second the catalan figure, and the third is George Hart's extension.

```
              Archimedean  Catalan    Hart's Extension

             d  dual
     oPxQo   a  ambiate    j  join    p  propeller
     xPxQo   t  truncate   k  kis
             t#            k#           Only verticies of type #
     xPoQx   e  expand     o  ortho
     xPxQx   b  bevel      m  meta
     sPsQs   s  snub       g  gyro    r  reflect
```

These operations are applied to the regular figures (TOCID), and also to the prisms Pn and Antiprisms An, and Pyramids Yn. The operators cumulate as prefixes, eg adC = ambiated(dual(cube)) = CO.

They produce a range of interesting polyhedra, but the operations do not extend well into four or higher dimensions.

Copycat

A number of polytopes that share the same mullitope or exposed shape, but different endotopes (internal crossings). [Bowers].

Cross polytope

Coxeter's name for the polytegum.

Cube

The usual name for the triprism, or hexahedron or {;4,3}.

Cube

Some authors regard the cube only as the Euclidean version of the {4,3}, and other versions of this are held to be hexahedra. In practice, any regular {4,3} becomes a cube under this definition if inscribed on the chordal surface.

Cubic

Triprismic. In non-metric systems, this refers to a column having 3 dimensions, not a mathematical operator: ie a cubic foot, acre or gallon is a volume that has the properties of a foot (side), acre (face) or gallon (volume).

Cupola

An exotic prism formed by two Stott vectors, that are radially outgoing: ie v(top)-v(base) has no negative elements.

CZ#

the complex cyclotomic numbers formed by the Z span of those numbers whose 2# power is +1.

CZZ

The set of all cyclotomic numbers CZ#.

Defect

Negative excess, used to measure an area of hyperbolic space.

Degree

A unit of angle, that 360 make a circle. This is notionally tied to the radian, although it is a circle-division that predates the radian by millennia. It is typically divided into minutes, seconds, and thirds. However, in base 60, one dispenses with theses names.

Degree Excess

A unit of solid angle, based on the degree, based on the notion that the solid angle is proportional to the excess of the angles that delineate it. The sphere is divided into 720 degrees excess. The function rad-> deg will convert sr-> deg E.

Degree of Freedom

A measure of how many independent variables a given presentation or polytope has. This is typically equal to the number of stott vectors needed for it, but non-Wythoffian figures have to be considered separately.

For example, a rectangular prism {}{}{}, a square prism {4}{} and a cube {4,3} all look the same, but are different presentations, and have different degrees of freedom. The rectangular prism has three (l, b, h), the square prism has two (b, h), and the cube has one (h). In four dimensions, the square-square prism {4}{4} and the cube-line prism {4,3}{} both look like tesseracts, but have two degrees of freedom. None the same they are different figures.

Degrees of freedom are given by the symmetry group and consumed by the size of the figure, etc.

This is what can add to a symmetry degree of freedom

- The number of marked nodes in the wythoff-symbol.
- Wythoff-snubs, such as in 3D, have n degrees of freedom, being alternate vertices of the omnitruncate. In general, Wythoff-figures have n degrees of freedom, and (n)(n-1)/2 variables to set. This is because the extra faces are simplexes: one can only define the n degrees of freedom of the omnitruncate, but the removal of the vertex figure produces a simplex with (n)(n-1)/2 potentially different edges.
- Coxeter-snubs have two degrees of freedom, being alternate vertices of a t{;p;q,r...}, q even.

The following reduce the degrees of freedom.

- Size. This may be realised as the curvature of the curcumsphere. For the circumsphere to become flat, the curvature of it must match space. Since an apeirotope is simply the surface treated as an N-1 dimensional tiling, to be flat defines a particular size, and hence a degree of freedom.
- Uniformity. Not all constructions can be made uniform, because there are more dependents than free variables. For example, the Wythoff-construct of using alternate vertices of the omnitruncate yields tetrahedra at the alternate vertices, this has six edges, but there are only four degrees of freedom available.
- Aspect. When Size is counted as a base quantity, the other degrees of freedom make for different aspects (or ratios). For example, the aspect of a picture might be 3:4. Expanding the diameter to 10 might make this 6:8, which does not alter the aspect. On the other hand, making the picture into a 2:5 ratio does make the image look flatter...
- Symmetry. Especially in comb-products of apeirotopes, one can "dispose" of surplus degrees of freedom by altering the symmetry group.

  For example, the tiling of triangular prisms has five degrees of freedom. The group is {/3/3/3:}{/oo/}. The five degrees of freedom are marked /. This gives rise to a tiling of 3D by hexagonal prisms where hexagons have alternate sides a,b or a,c or b,c, and the heights of the prisms alternate in layers of d and e.

A check on the number of degrees of freedom of a given presentation does not reveal that it is impossible. All it shows that a solution may be extraneous. One can construct a laminatruncate on any lt{;p;q,r} where {p,q} is spheric, and {q,r} is hyperbolic. Sascha Rogman constructed a tiling of right truncated icosahedra as a lt{;3;5,4}, but this is not uniform. The only uniform example is lt{;4;3,8}.

Degrees (square)

The notion that a degree is a length on the surface of a sphere, and that an area can be defined in terms of a square of length 1 degree. The function rad->deg will convert sr -> degE -> sqDeg

Deltahedron

A convex polyhedron bounded by triangles, usually equilateral. This concept can be generalised to a polytopalotope

Density

A notional multiple covering of a surface by d flags. A ray from the centre to the exterior passes through d flags. Note that some flags may be inwards-pointing.

Di-

Two, when taken as a pair. so dyad, diameter diface angle. It carries the connotation of "between"

Discrete

A kind of infinitely dense where one can show that a point is not a member. For example, the decimals are discrete, since 1/3 can not be represented as a decimal.

Ditope

A polytope with two faces, back to back. This is the result of a line prism of zero height. A ditope is solid if the prism with non-zero height is solid. A pentagonal dihedron is solid in 3D but not in 4D, since a pentagonal prism is solid in the former but not in the later.

Dodecahedron

The fifth platonic polyhedron, and the seventh uniform polyhedron. This figure is bounded by 12 pentagons, and it has 30 edges and 20 vertices.

The extended meaning of the dodecahedron refers to figures reducible to the form {x,3,3,3,3...}, here, x=5. The gossettododecatopes are an example of this.

Drift

A measure of an exotic prism, which measures the projection of the lacing in the base. This is calculated from the difference in Edge vectors from the top to the bottom. The right angle triangle Drift^2 + Height^2 = Lacing^2 is used to find the height of the exotic prism.

Dual

A topological process, where each surtope is replaced by its orthosurtope. For an n-d polytope, this replaces an n-1 surtope by an N-n surtope.

duoprism

G Olshevsky's name for a prism not reducible to a slab prism. Such occur in four dimensions as a polygon-polygon prism.

duotegum

The name G Olshevsky uses for the tegum product of two polygons. This was formerly called "duopyramid". It is interesting to note that John Conway contrasts "prism" and "pyramid" as well, with the hyper- prefix.

Dyad

A pair of points that are considered to be the surface of a 1-sphere.

Dynkin Matrix

The presentation of the Dynkin Symbol as a matrix. a_ii = 2 always, and a_ij = -2cos(x_ij), where x_ij is the angle between the i and j nodes: ie a_ij = -2 cos(pi/p).

Dynkin Symbol

The presentation of the fundamental region of a mirror-group as its dual. This makes the region-walls into points (nodes), and the margins into the lines (branches). The branches are marked with a number p, representing a wall-angle of 180°/p. Where p=2, the branch is neither marked or drawn. Where p=3, the branch is drawn, but not marked. Nodes are then marked with the appropriate activities associated with the that mirror.

E1

A scale based on measuring the chord subtended by the ends of an edge. It is more commonly used than the true-length.

E2

The square of E1, very commonly used.

#-Edge

An edge is a 1-dimensional surtope: that is a line part of a polytope connecting vertices. An #-edge is the generic name for a n-dimensional surtope.

Edge vector

A presentation of the edges of a Wythoff-figure, in the order of the symbol, which gives the measures of the edges perpendicular to that given mirror.

For example, in the group o3o5o, the edge vector of x3x5o is (1,1,0). There is no need for them to be integral or even positive. (see drift)

Efficiency

A measure of the fraction of space occupied by spheres centred at points in space, usually the vertices or cells of an apeirotope, or the stations of a reflection group. Several units are used for this:

- Q-Unit = nr of spheres of diameter r2 into a unit prism-measure
- Leach Unit = nr of unit-radius spheres into a unit prism-measure
- Implied S = indicated solid angle of simplex in tegum-radians
- fraction of space = percentage of all space taken.

Endo-

A prefix used to indicate that all the interior crossings of the faces of a polytope are represented on the surtopes. In practice, the edges of the pentagram are not regarded as crossing, and one can only see one edge or corner at a time. The fait of density prevents one from seeing more than one.

Endocell

The polytope, reduced to a single plane, with all of its surtopes marked solid. In practice, this is the normal representation, since we have no meaningful way of showing the layers of density.

Endoface

A face with all of the crossings marked. However, this goes only as far as the actual faces of the polytope go, not the complete intersection of all face-planes on a given face-plane. One might show the exposed sections, or mullifaces on such a diagram.

Endofy

To draw in all crossing faces. This is the usual mode for line-drawings.

Endotope

A surtope, with all crossing surtopes marked on it.

Etching

An apeirotope that forms on a face of a packed laminatope. This is usually formed by coplanar surtopes of the interior polytopes. An example of an etching would be the hexagons on the surfaces of a layer of hexagonal prisms.

Eutactir star

A set of vectors radiating from a point, used to construct mirror-edge and other figures. For any mirror-edge figures, the eutactic star consists of vectors perpendicular to the different mirror-planes, and centred on the centre. The (0-1) span of these vectors produce zonotopes. In the sense that when these are pair-wise orthogonal, the result is a polyprism.

Excess

The measurement of 3-angle by the excess of the spheric polygon over the plane polygon. The total of the sphere is two circles excess.

#exon

A 6-edge, or a polytope so bounded by 6-edges (ie a 7-tope)

exoskeleton

The outer shell or visible bits of a polytope. see mullisurface &c. Note that the mullicell aslo considers the interior as something that could be made of wood or clay, not just a surface.

Exotic Altitude

The space orthogonal to the exotic transverse. The bases project into this space as points, and lacing edges as lines. Each vertex in the exotic altitude represents a different vertex node.

Exotic Drift

The difference of the Stott vectors of the top and bottom bases of an exoprism. This is used to calculate the circumdiameter of the exotoprism.

Exotic Mode

A designation for {np/nd} as n superimposed copies of {p/d}. The sense here is that the resultant polygon is an exotic polytope.

Exotic Polytope

A polytope with coincident surtopes. For example, an exotic hexagon might have opposite vertices coinciding, such as two triangles joined at a vertex.

Exotic Prism

A figure formed by two figures of the same symmetry, placed in parallel planes. The class was investigated as the vertex figure of a polytope of two degrees of freedom are exotic prisms. Accessing these figures is a way of calculating the Stott Matrix.

Exotic Pyramid

A form of exotic prism, which has three or more bases, arranged as a pyramid. The vertex of all mirror-edge figures are exotic pyramids. The normal pyramid product can be regarded as an exotic pyramid with orthogonal bases.

The Exotic 'product' resembles the pyramid product, and is equal to it, when each pair of bases are completely orthogonal to each other.

Exotic Tegum

The figure formed by the common intersection of two pyramids of different bases of the same symmetry, placed so that the apex of each is in the base of the other. This is the dual of the exotic prism. Where the prisms do not span all space, the whole of the orthspace is taken. For example, the exotic tegum formed by two lines is the intersection of a pair of planes intersecting parallel to the y axis, and a second pair intersecting parallel to the z axis.

Exotic Tower

A stack of exotic prisms, that share the same axis. The vertex-first or face-first sectioning of a polytope treats the figure as an exotic tower.

Exotic Transverse

The space in which the combined symmetries operates across. The wythoff construction of all the figures of the bases in their stated orientation spans completely the exotic transverse.

Exotic Vertex

The projection of the individual bases into the exotic altitude produces a single vertex. For example, a cupola has an exotic altitude of a line, and the two bases project onto this as two exotic points.

exotope

The mullicell or polytope with the same exposed surface. Mullitope also includes the interior as a single-density layer.

exotoprism

An exotic prism. The bases are described in order, eg a triangle hexagon exotoprism = triangular cupola.

exotegum

An exotic tegum The bases are described in order, eg a triangle hexagon exototegum.

Expand

An operation that moves faces radially outwards, producing a series of prisms of the surtope and its orthosurtope. Such an operation is also called the runcinate.

f-unit

An angle represnting 1/14400 or 0:0001 of a sphere, being the symmetry of the group 4f = [3,3,5]

Face

An n-1 edge; specifically, one that bounds a polytope. A face is a solid portion of a plane.

#-Face

The sequence of faces decending from the polytope. This is an N-n edge, as follows: 0=cell, 1=face, 2=margin, 3=nanol, 4=picol, 5=femtol, 6=attol, 7=zettol, 8=yoctol. [thousands rule]

facelet

facelet Any small face, such as produced by some operation, eg corner fractalisation, or mullification. An exposed fragment of a face.. see mulliface.

facet

face This is used when face is used to mean surhedron, or 2-edge. The redefinition avoids constructions like "the facets of the 120chora has pentagonal faces"

#-faceting

A process involving passing interior planes through the vertices of a given polytope. This produces a series of new faces. The process is a dual of the #-stellation. When #- is applied, it implies that the #-margins are shared between the faceting and the figure: eg a great dodecahedron {5,5/2} is a 1-faceting of the icosahedron.

fractionally infinite

A kind of infinity formed by the non-closure of multiplying by fractions. For example, the complex number 0.6+0.8i forms a fractionally infinite division of the circle.

Fractional infinities are characterised by relatively localised finite structures, and are topologically equal to hyperbolic honeycombs involving infinite cells.

femtol

A 5-face, or N-5 edge.

fi

The number (1+r5)/2 = 1.61803398875 = 1:7419 8287 V8V3 43E0

Flag

A flag of a polytope is a pyramid formed by the centre of a face, and the flag of the face. This means that the flag of a polytope is the pyramid-product of the centres of the 0,1,2,...,n-1 edges. The existence of flags imply that regular polytopes must have faces that can be regarded as solid.

Flat

Not solid. The connotation here is that it is solid in n-1 space, that is, the orthospace of a line.

#-Flat

A #-space less than all-space, and part of it, having the same curvature as it. A 0-flat is a point, a 1-flat is a line, an n-1 flat is a plane, and an n-flat is a realm.

General

The 1-army leader, with which, other army members share the same vertices.

genus

A description of a 2D surface according to how many holes it has. Polytopes marked up on such a surface have a defect in the surtope polynomial, which is numerically related to the number of holes a solid has. In higher dimensions, the concept of genus is replaced by the hole polynomial and surtope defect. In 4D, figures with the same surface topology can have different genus: see the discussion on holes and torus.

#-Globlutope An n-sphere, being the limit of a polytope as the faces become tiny. This is the locus of points equidistant in n-space from a point. A 2-globlutope is a circle, a N-globlutope is a sphere. People who don't like this construction might want to use #glome.

#glome

This term is an alternate for the n-sphere, for those who find the 'tiny face' approach unappealing. A tetraglome is a 4-sphere, and a triglome is what we in 3D call a sphere.

#gon

This prefix actually refers to the corners, not the edges of a polygon. Arguments for making this into the sense of "edge" rather than "vertex" centre around the behavior of "elbow/ell" and "cube/cubit".

Gonglotope

A glomotope and its interior. An N-1 glonglotope is a disk, an N-glonglotope is a ball. This is a second-series name, in practice.

#gongyl

[G Olshevsky] A solid disk or ball.

Gosset figures

Figures derived from the third trigonal series. Th. Gosset described a series of semiregular figures, being a vertex-figure series based on the triangular prism.

Gossetododecatope

The gosset figure with a vertex with simplex-vertex symmetry. These are of the form of 1_k2 or {G,3.....}. These are the largest of the gosset figures.

```
5D  1_21   gossetododecateron   {G,3,3,3}  = half-cube
6D  1_22   gossetododecapenton  {G,3,3,3,3}
7D  1_32   gossetododecaexon    {G,3,3,3,3,3}
8D  1_42   gossetododecazetton  {G,3,3,3,3,3,3}
9D  1_52   gossetododecayotton  {G,3,3,3,3,3,3,3} = apeiroyotton
```

Gossetohexatope

A name suggested for the half-cube in n dimensions. The reasoning being is that all polytopes of the i_jk class have names in gosseto-

Gossetoicosatope

This is a series of polytopes that have the previous dimension as an vertex figure. The three-dimensional representative is the triangular prism or gossetoicosahedron. In six, seven and eight dimensions, these figures have a symmetry distinct from any of the regular figures in that dimension.

```
3D  gossetoicosahedron   X_21 {3,B}          = triangular prism
4D  gossetoicosachoron   0_21 {3,3,B}        = rectified pentachoron
5D  gossetoicosateron    1_21 {3,3,3,B}      = half-pentaprism
6D  gossetoicosapenton   2_21 {3,3,3,3,B}
7D  gossetoicosaexon     3_21 {3,3,3,3,3,B}
8D  gossetoicosazetton   4_21 {3,3,3,3,3,3,B}
9D  gossetoicosayotton   5_21 {3,3,3,3,3,3,3,B}  [apeiroyotton]
```

Gossetooctotope

This is the gosset polytope that has a half-cube vertex-figure, and is therefore of the form 2_k1.

```
5D  gossetooctateron    2_11  {G;3,3,3}      = pentategum
6D  gossetooctapeton    2_21  {G;3,3,3,3}    = gossetoicosapeton
7D  gossetooctaexon     2_31  {G;3,3,3,3,3}
8D  gossetooctazetton   2_41  {G;3,3,3,3,3,3}
9D  gossetooctatotton   2_51  {G;3,3,3,3,3,3,3}  [apeirogon]
```

Grand

The result of 3-stellation, or surchoral stellation.

Great

The largest of a series of figures, usually in contrast with small and other prefixes.

Great (stellation)

The result of 2-stellation, or surhedral stellation.

ha

The shortchord L1 of the heptagon, = 1.80193773580 = 1:9627 V848 V769 56V7

Hatch Loop

The presentation of a Schwarz-polygon as a Dynkin symbol. Unlike the normal loops in the Dynkin symbol, non-adjacent mirrors do not cross. In a normal loop, they cross at right angles.

Don Hatch gives examples of this at his web-site.

hb

The long chord of the heptagon = 2.2469796037 = 2:2976 6090 7526 8379

#hedron

A 2-edge When used with a prefix suggesting a number of 2-edges are implied, this implies a three dimensional figure, eg dodecahedron = 12 hedra.

heptagon

A regular polygon with seven sides, the smallest not to be constructed by classical methods (ie compas and straight-edge).

```
    chords
        shortchord   1.801937735804  = 1:9627 V848 V769 56V7  = ha
        longchord    2.246979603717  = 2:2976 6090 7526 8379  = hb
    diameter
        circumdiam   2.304764870962  = 2:3668 7383 7716 05v9
        indiameter   2.076521396572  = 2:0921 V8E6 9522 7826
        d2 measure   5.311941110422  = 5:3751 E428 78V4 5966
    area
        side         3.633912444001  = 3:7608 4084 4669 76V6
        indiameter   0.842755582913  = 0:V115 8177 8089 4986
        circumdiam   0.684102547159  = 0:8211 0924 2157 70V0
```

heptagonal flat

The heptagon version of the pentagonal fibonacci series. This is an iterative series that spreads over a plane. In one sector, the numbers converge to the shortchord and longchords of the heptagon. Here is a sample of a small region near the origion. As in the pentagon, this may be used to find powers of a^m b^b

```
   a^n     -2  1  0  1   2   3   4   5    6   How to make the series grow
   b^m    ---------------------------------          z-y    y-x
     -1 | -3 -2  0 -1   1  -1   2  -1    4    x+y-z  x       y   x+z
      0 |  3  1  1  0   1   0   2   1    5      z-x  z      z+y
      1 | -2 -1  0  0   1   1   3   4    9      x+y x+z+y
      2 |  2  1  1  1   2   3   6  10   19
      3 | -1  0  1  2   4   7  13  23   42    When  x,y,z =
      4 |  2  2  3  5   9  16  29  52   94      (1,0,0)  units flat
      5 |  1  3  6 11  20  36  65 117  211     (-1,1,1)  symmetric flat
      6 |  5  8 14 25  45  81 146 263  474      (3,1,2)
      7 |  9 17 31 56 101 182 328 591 1065     (1,ha,hb) lograthmetic
```

In the units-flat, the power of a^n b^m can be read straight from the table since n,m points to x in x+ya+zb, so a^5.b^3 = 23+42a+52b. This mirrors the use of the phi^n = F(n-1) + F(n)phi.

When x,y,z assume the lengths of the chords, the resultant flat is perfectly logarithmic.

Heptagonal Integers

The Heptagonal Integers are the set of numbers of the form x+ay+bz, where x, y, and z are integers, and a, b are the short and long chord of the heptagon. This is the Z-span of the chords of the heptagon.

The set is closed to multiplication, since a*a=1+b, a*b=a+b, b*b=1+a+b, but not to division. A normal prime decomposes into lesser factors if it is equal to 1 or 6 mod 7. The cube of the number -1+a+b is 7ab.

Hole

The sense of hole is that of a throughle, and not a pit or depression. A hole is a missing surtope, that when placed, prevents the formation of any shell that can not be made to vanish without crossing the surface.

The location of the hole is in the same surtope or region as the non-vanishing shell. This may be on the interior, exterior, or in any of the surtopes.

A one-dimensional hole is a line with its interior missing. This gives a pair of points. Such can be useful for testing disjoint regions. The missing surtope would fill one of the regions.

Hole, extended

A polytope may contain a sequence of surtopes that by right ought bound in some space, but does not bound. This can not be detected by the loop test, but makes itself known in the surtope polynomial.

hole polynomial

A polynomial that represents the dimensionality of the holes at zero and infinity. One can represent these separately, usually the hole-polynomial is taken to be the hole-at-infinity form (since the hole-at-zero form is readily calculated by replacing a^(n+1) with a^(N-n).

honeycomb

Apeirotope. The term refers to the section of a bee's honey comb, which resembles a hexagonal tiling in 2D. In practice, it's a tiling of a slab with hexagonal prisms, capped by a rhombic dodecahedral cap.

Horo-

A prefix that relates to a curve of zero curvature. In Euclidean space, such is flat, but in hyperbolic space, this is a curved space, flat is negative curvature.

horopoint

A point at the centre of a horocycle, or normally on the horizon. Lines that contain a common horopoint are parallel in that direction.

horosphere

A surface that is perpendicular everywhere to lines that are parallel in a common direction. Such a surface has euclidean geometry. Horospheric distance is the length of an great arc on a horosphere. It can be treated in the normal euclidean geometrical sense, as E1 and E2 attest.

horosurtope

A horotope as a surtope. Such has a horopoint at the centre, and lines passing normally through the surface are parallel in the interior of the figure.

horotope

A polytope whose surface generally follows a horosphere, or surface of zero curvature.

Hotel

A name used for the polygonal polycomb. see Comb Product

hyper-

Over. Can mean the sense of over 3 dimensions, or as a contraction of hyperbolic. It is for this reason that this prefix is avoided. ~cube = tesseract, ~plane = 3-flat. ~sphere = globluchoron or tetraglome.

Implied s

A unit of honeycomb efficiency. The value s here is the solid angle of the simplex, measured in tegmal radians. The current limits on s are 1 and sqrt(n/[e=2.718]) tegmal radians. In a tiling of spheres with only simplex holes, a simplex of edge 2 is occupied by (n+1) simplex angles, meaning that the maximum efficiency can not be more than sxsqrt(2^n)/sqrt(n+1). In 8 and 24 dimensions, the maximum efficiency implies a value of s>1.

In

A point that is part of the solid space of a figure, and is part of the space that the solid occupies in that space, is in the solid. For example, for a polygon in 3D, the only points in the polygon are those that form the interior based on the plane containing the polygon.

Infinigons

A polygon with infinite number of sides. Since the side can not be used to tell two apart, the pattern is to use the square of the shortchord, such as {w3.61803398875}. Infinigons can be of any curvature, the numberline or parabolic infinigon is {w4}. Infinigons have an interior, but may be also treated as apeirotopes.

Infinitely dense

A generalised term to mean that the density is without bounds. For our purposes, it is meaningful to describe the nature of this density.

- **Discrete** means that it is easy to show that points are not in the set: eg the decimals are discrete, because 1/3 does not come out as an exact decimal. None the less, there are points as close as one needs, and closer.
- **Fractional** means that the density is a result of multiplying by successive fractions; and therefore needs either a multiplicative operator or an infinite number of additive operators.
- **Integrally** means the sort of infinitely dense arising from the integer-span of a closed set of irrationals, such as the chords of a polygon. The order of an integrally infinity is equal to the number of independent generators required.

Infinitope

A polytope with infinite number of faces. Unlike the apeirotope, this has an interior. Thus the infinihedron {6,3} has infinitely many hexagons as faces, and a solid interior in 3D, one side of the {6,3}'s surface. It is counted as a 3D figure. The apeirotope {6,3} is just a tiling of 2D space, by hexagons. Such a figure tiles all space, and there is no "interior", except that of the hexagons.

```
   (no space)            . . (empty space) .      .(empty space).
                        . . . . . . . . . . .    . . . . . . . . .
---o-----o-----o----    ---o-----o-----o-----    ---o-----o-----o--
                        +++++++++++++++++++++    ++/+\+++/+\+++/+\+
   (no space)           +++(solid interior)++    +/+++\+/+++\+/+++\
                        +++++++++++++++++++++    o-----o-----o-----
                        +++++++++++++++++++++    . . . . . . . . . .
  apeirotope               infinitope              laminatope
```

Infinity

A number taken to represent the inverse of zero. The geometric infinity or radius of space appears to be specific, though unapproachable. On the other hand, it is possible to reach transfinite numbers, which appear to be smaller. The normal presentation is the number of faces that a {w4} has, or the radius of a horocycle [both the same thing].

Infinity, at

The surface at infinity is taken to be a surface that represents direction. Two lines approaching the same point at infinity are parallel. Under inversion, the point at infinity becomes the point at zero. Holes at infinity are holes outside the figure. See also zero, at.

Inversion

A transformation of n-space around a point, so that a point at r,A transforms into 1/r, A. A is understood to be a generalised n-D angle direction. Spheres, including planes, transform into spheres, including planes. A surface is flat if it contains the centre of inversion (or point at zero) on its surface. The inversion model of euclidean geometry can be used in any given geometry, the relative economy of constructing lines perpendicular to a ray (a straight line containing the point at zero), is considerably easier than constructing either a general line through two points, or any other isocurve passing through three points.

Inversion Dual

A curious feature, where, the midpoints of the surtopes of a polytope lie on the spheres containing the point at zero. For example, the same 62 points, being the 12 vertices, 30 edge midpoints and 20 face centres of an icosahedron, lie on 12 spheres that include the point at zero. These twelve spheres also contain all 62 points of the icosahedron, which, on inversion, become a normal dodecahedron.

Iso-

Equal. Here used for the terms derived on the isosequence, and isospace.

Isocurve

A surface of uniform curvature, such as a sphere, horocycle, plane. Isocurves also include "flat" surfaces, which are specific examples of isocurves with the same curvature as all-space.

Isoflat

A surface with the same curvature as its all-space: so also, isoline, isoplane. The term can also be used when other definitions of "straight" are in force.

Isopower

A value k^^n = t(n), where t(n) is an isosequence in k, and t(0) = 2, and t(1) = k. In any isosequence on k, t(n) x k^^a = t(n-a) + t(n+a).

Isosequence

A sequence t(n) such that t(n)*k = t(n-1)+t(n+1). Such is an isosequence on k. Sines and Consines of multiples of angles form isosequences, but the term is more general than this.

Isospace

The space that isocurves are drawn in. This is usually taken to be all-space.

Join

This is an operation described by Professor John H Conway as the set union of vertices. When the join is complete, the centres of the figures coincide and the result is a tegum. When the operation is not complete, the centres are displaced, and the result is a pyramid.

While the join nicely complements the cartesian, in that two products are merged into one, the idea is that one ought treat these as separate products.

The Conway-operator that gives the strombiate direct. This is the dual of the Expand operator.

Klein Projection

The Azymthal projection of hyperbolic geometry. Like the spheric version, straight lines continue to be straight. All hyperbolic space is projected onto a disk with a finite radius. Circles are still circles, horocycles, or equidistant curves, as they share zero, one or two points with the edge of the disk. The projection is not as esthetic as the Poincaire, since features towards the edge are heavily compressed radially.

L1

The ratio of the shortchord to the edge of a polygon.

L2

The square of L1. This is regarded as the most significant number describing the polygon, some of which are only ever described in terms of L2. See also w.

Lacing

The edges of an exotic prism or segmentatope, that have a vertex in each base. The term is inspired by the zigzag around the perimeter of a polygonal antiprism, which resembles lacing holding the ends of a drum.

Lamina

A layer. The sense here is the alternating layers of triangles and squares in the non-Wythoff honeycomb.

Laminatope

A polytope bounded by unbounded faces, for example, a layer or strata. The sense is seen in a layer of triangles or squares of an apeirogon antiprism or prism.

```
   (no space)                 (empty space)          (empty space)
                           . . . . . . . . . . .    . . . . . . . . .
---o-----o-----o----       ---o-----o-----o-----    ---o-----o-----o--
                           +++++++++++++++++++++    ++/+\+++/+\+++/+\+
   (no space)              +++(solid interior)++    +/+++\+/+++\+/+++\
                           +++++++++++++++++++++    o-----o-----o-----
                           +++++++++++++++++++++    . . . . . . . . .
  apeirotope                  infinitope              laminatope
```

Laminatruncate

Where a truncation yields a laminatope, the unbounded faces may be used as mirrors, to full all space. This is normally applied in 3D hyperbolic honeycombs, eg laminatruncate {4,3,8} fills all space with truncated cubes.

Leaf

The second-series name for a face, or sesquihedron

Leech unit

The fitting of a sphere of diameter 2 into a unit-edge measure-polytope. The efficiency of the Leech lattice in 24 dimensions is 1 leech-unit, or 4096 q-units.

Maps, absolute

This is a series of maps, that can project one geometry onto some other surface. Because non-euclidean geometry has a definite size, these techniques are also important in the same geometry, if one wants to reduce size. Maps can be point or line-centric.

```
  SPHERICAL          HYPERBOLIC         EUCLIDEAN    preserves

 RADIAL
  stereographic      poincare model *   inversion    isocurves, angles
  azythmal           klein model *      (nature)     straight lines
  orthogonal *       orthogonal         (nature)     area

 TRANSVERSE
  mecator            [mecator] *        (nature)     preserves lexidromes
  cylindrical *      [cylindrical]      (nature)     preserves area
```

maps, radial

This is a transform of points around a given point, in a reversible way. Different mappings can be defined.

```
unit map       R, A  ->  R, A
centre map     R, A  ->  -R, A
inversion      R, A  ->  1/R, A
```

Marek Snub

A H2 snub-operation that reflects a snub sequence, as follows.

```
normal snub    A132132 A213213 A321321
marek snub     A121323 A312132 A231213
```

```
   o 1 o       o 1 o       o 1         o 1 o       o 1 o       o 1 >1
  2     3     2     3     2           2     2     2     2     2    >2
 o       o 1 o       o 1 o           o       o 1 o       o 1 o     >1
  3     2     3     2     3           3     3     3     3     3    >3
   o 1 o       o 1 o       o 1         o 1 o       o 1 o       o 1 >1
 2      3     2     3     2          2      2     2     2     2    >2
o        o 1 o       o 1 o          o        o 1 o       o 1 o     >1

        Regular snub                       Marek-snub
```

#-Margin

The surtope that separates the faces of a polytope. An #-margin is an N-#-1 surtope.

Matrix-dot

A matrix-operation applied to two vectors: the first vector is multiplied by the matrix, and the result dotted with the second vector: ie the matrix dot of V and W is sum(i,j) (S_ij V_i W_j). Matrix dots are used when the defining vectors are not at right-angles to each other.

Matrix-norm

The matrix-dot of a vector and itself.

Measure

The notion of measuring things suggest that an item can be moved about the space, and placed beside separate cases. What holds for time applies here as well. Units of measure of content are the polyprism, polytegmal, polyglomal units.

Measure polytope

A polyprism. See #prism. This figure is usually rated the first of the uniform polytopes, and has a special name: square, cube, tesseract, &c.

Mecator projection

A representation of space, which produces lexidromes. The base projection for this is the equator, drawn in nature, and the lines of longitude, perpendicular to the equator. A lexidrome is a line that cuts every line of longitude at a given angle. In the case of the hyperbolic mecator projection, the map has a finite width, as the lexidrome converges on some line of longitude.

Mete-star

The compound of 120 pentachora that have the same face-planes of a 600chora, and the vertices of a 120-chora. The name comes from the fact that 4-angle is measured in the symmetries of the pentachora and the 120-chora. This compound, and its two daughter compounds, do not belong to the standard constructions of compounds in four dimensions. See also s-unit, f-unit.

The mete-star is considered to be one of the few euclidean examples of compounds more common in the hyperbolic world eg: {7,3}[24{14,7]{3,7}, or the euclidean tiling {4,4}[36{4,4}]{4,4}.

Mirror

A surface that does inversion.

Mirror-edge

A polytope whose every edge is perpendicularly bisected by a mirror. Many, but not all, of the mirror-edge figures can be constructed by Wythoff's construction. The exceptions are those in non-simplex mirror-groups.

There is no requirement that the edges of a mirror-edge figure be equal. Any rectangle is a mirror-edge figure.

Mirror-margin

A polytope whose every margin lies in a mirror-plane of the figure. This is the dual of the mirror-edge figure. A mirror-margin figure can be constructed by converting some mirrors into margins. The face then becomes a reflection of the mirror-margin in different faces.

There is no requirement for the dihedral angles to be equal. Any rhombus is a mirror-margin figure.

Mobius

This name is applied to mirror-groups based on something other than simplex or simplex-prisms. The group is quite common in hyperbolic space.

Mobius Dynkin Symbol

An adaption of the Dynkin Symbol, designed to reflect the Mobius polygon. It can be distinguished by a leading loop-node. Branches represent angles pi/n. Unconnected nodes represent sides that do not intersect. This is also called a Hatch Loop and the leading node a Hatch Node.

Mobius Group

A reflective group, where the fundamental region is a polygon with more than three sides.

Mobius Mirror (edge/mirror)

A figure based on a mirror-edge construction on a mobius group. In H2 tilings, the class is super-infinite.

Mobius Omnitruncate

A pseudohedron resulting from placing a vertex in the interior of a mobius group.

Mobius Snub

A generalised snub, based on alternate vertices of a mobius omnitruncate.

Modprism

A semiate truncation of a prism, retaining vertices whose ordered stations add to zero, in some modulus. See 'semiate', 'step'

Modtegum

A semiate stellation of a tegum, retaining faces whose ordered stations add to zero, in some modulus. See 'semiate', 'step'

Motion

The notion of motion is resolved into identifying similarities in a sequence of different geometric situations. The snapshots of motion are really different geometric situations, the equality of which needs to be shown. (Moving a square on a Mecatour projection will reduce the area the more it is moved away from the equator).

mounted

The idea of "mounted" is the same as one might mount something on a wall, &c. A polytope is mounted, that, if it shares any of the interior of some surtope with another polytope, than the whole of that surtope is a surtope of both polytopes.

Mullet

The heraldic name for the shape representing the outline of the pentagram. When one needs to sew five-pointed stars, one cuts out mullets. This term is used also for four and six pointed stars, with and without an interior hole. Whatever the ultimate meaning, the sense of something cut out of the fabric of space is preserved in the mulli- prefix.

Mulli-

A prefix used to designate the polytope wrought from the fabric of simple space, that has the same occupancy as a given figure. See mullet for the derivation of this prefix. See also Endo-

Mullicell

The figure in simple space, that has the same "outline" as a second polytope. Note that the mullicell also has an interior bereft of internal markings, and suitable of being filled with wood or clay or whatever. In the case of facetations and setllations, this figure can be quite different. eg, the mullicell of the stellated dodecahedron is a figure with 60 triangular faces, 90 edges, and 32 vertices, this can be made by "glueing" pyramids to the faces of a dodecahedron.

Mulliedge

The edge of a mullicell. Mullification may make several mulliedges out of an existing edges, and additional mulliedges where faces intersect at a non-edge. For example, the dimples in the mulliform of the great dodecahedron have mulliedges not derived from the original edge, and the edges of the stellated dodecahedron produce three mullidees a piece.

Mulliface

The face of a mulliface. There may be several different mullifaces in the same plane.

Mullify

The act of removing the interior lines of the polytope, leaving just the outline. This is then remade into a new polytope with additional vertices, edges.

Mulligroup

A group or bund of different polytopes that share the same outline or mullicell. Polytopes in a mullicell bund are copycats.

Mullileader

The notional leader of a mullicell bund.

Mullisurface

The complete surface or exposed exterior of a polytope.

Mullivertex

The vertex of a mullicell. While these include the vertices, there are additional ones formed by the crossing of edges, and the triple crossing of surhedra, etc.

nanol

The 3-face or N-3 edge. third-margin, or margin's face. This is also from the thousand-rule.

Navy

The dual of the Polytope Army, members common to the same navy unit share the same surtope, and those of higher dimensions. Because the lesser dimensions are relocated, the sharing of surtopes implies only that there is one in the corresponding flats.

The navy units are closely allied to the process of stellation.

```
 Unit         group     leader         Stellation        name
-------------------------------------+---------------------------------
 face-navy     fleet     admiral   |   edge-navy         stellated
 margin-navy   ship      commodore |   surhedron-navy    great
 2margin-navy  boat      coxwain   |   surchoron-navy    grand
```

Navy units form sets etc as does the army units. However, increasing the surtope of the navy unit makes the grouping coarser, not finer. So while an vertex-army contains several edge-armies, the edge-navy contains several vertex-navies. See also Army

Node

The second-series name for an vertex: see sesquitope

non-Wythoffian

The designation given to a uniform figure not constructable by Wythoff's mirror-edge construction on a simplex-mirror group. There are not many of these outside of the hyperbolic apeirohedra.

```
 Name             Description             Discoverer
 snub {;P;Q;R:}  vf (3,P,3,Q,3,R)         ?
```

s{;3;4,3}	snub 24-chora	Wythoff, Stott
j5j2j5j	grand antiprism	John Conway & Richard Guy
s{;3;4,3,3}	snub {3,4,3,3}	?
t-pr	prismatoreflects*	?
borromeal	{p}-borromochora*	Charlie Gunn
(four cases)	octacubics*	Wendy Krieger
pt{3,5,3}	partialtruncate*	Wendy Krieger

See also snub(Coxeter) and snub(Wythoff)

Nulloid

The notional -1 edge, that arises as the dual of the solid substance of a polytope. This is normally residing at infinity, and gives rise to "spiritual" connections because of this. It is part of the surtope equation.

Noid

Coxeter's name for any spheric tiling that in parabolic space becomes flat. Typically, these involve a 2 in the Schlafli symbol, and marks on only one side of it, eg {;2,p} = hosohedra, {p,2} = dihedra.

A noid is **solid** if the sphere that the tiling covers is solid.

octagonal-cubic

A group of four non-Wythoffian hyperbolic apeirotopes that share a lot of symmetry with {3,4,3,8} and its sections.

- The laminatruncated lt{;4;3,8}. This normal truncate has faces {3,8} that are smooth. Such faces can be used as mirrors, to have space completely filled with t{;4;3}, the vertex-figure is the octagonal tegum formed by rotating an octahedron 45° [15:00] around an axis. The edge corresponds to a {3,8}. The symmetry group contains the {8,3,4}, formed by one of the octahedra.
- The extendotruncated xt{;4;3,8,2} is a tiling of bt{3;4;3}. The vertex figure of this is an bi-octagon tegum, any octagon by a side of the other side is the vertex figure of a {3;4;3,8}. The edge corresponds to a {3,8}.
- The dual of the xt{;4;3,8,2} is xt{;2;8,3,4}, a tiling of bioctagon prisms, 288 [248] to a vertex. The vertex figure is o3m4m3o, the convex hull of dual 24-chora. The edge is that of {4,8} = {8,6}
- The o8o4xp3xr. Without the p and r, the vertex figure of this is a distorted octagon prism xq8ooq, the top octagon being of edge 1, and the base and sloping edges being of edge r2=1.414. The trapezium sides are the vertex figure of the rhombocuboctahedron {;4,3;}, the top and bottom being {3,8} and {4,8} respectively.

  the effect of the p operator is to replace the top with a pyramid of sloping edge r2, representing triangular prisms. The r operator reflects this in the base, giving a figure bounded by 16 triangles and 16 trapezia.

  the vertex figure consists of the two poles, and 24 vertices being the cuboctahedral vertex figure of {8,4,A} and the same rotated 45 degrees [15:00] around one of its square axies. The equator and the lines of longitude are all octagons edge r2, the two lines of latitude is an octagon of edge 1.

Octagonal Ball

The vertex figure of the o8o4xp3xr is an exotic tower oxqxo8ooooo&q, this has 32 faces. It looks like a globe, with lines of longitude and latitude at every 45 degrees.

Octagonal Barrel

A figure made from the intersection of a rhombocuboctahedron, and the same rotated 45 degrees (0:15 circle) around a square face. Such is the dual of the octagonal ball, and tiles H3 as o8o4mpAmr.

Octogonny

The special name for the bitruncated 24-chora, bt{3;4;3}. This has 48 truncated cubes as faces. This figure discretely tiles 4-space.

Octagrammy

The quasitruncate 24-choron bt{3;4/3;3}, having a density of 73 and 48 quasitruncated cubes as faces. This figure tiles 4-space discretely.

Off

A point not in some all-space. Points off a space are neither in or out of elements of the surface. For example, a point may be off a line, plane.

Omnitruncate

A figure with a vertex in the interior of every flag of the source.

On

A point is on a surface, if it is a part of a surface which is a proper subset of all-space.

orthogonal projection

This is a point-centric projection, where the circumference of circles centred on a fixed point is preserved. In both the Euclidean and Spheric geometries, this is the result of projecting the thing from afar onto a plane. Hyperbolic circles tend to grow very fast in terms of the radius, the area of a circle divided by the circumference can not exceed a fixed amount.

Ortho-

The meaning here is that it is in a space orthogonal to a given figure. The sense of "right-angled" is abandoned...

Orthorealm

The N-n space orthogonal to an n-space. The implication here is that the realm is infinite in extent.

Orthospace

The N-n space orthogonal to an n-space. This also implies that only a part of "all space" is being considered. The orthospace of a point is all-space.

Orthosurtope

The surtope of a dual that corresponds to the surtope of the figure. This is the dual of the surtope-figure.

Orthotope

This term is already in use for polyprism, and we are not planning to pre-empt it just yet.

van Oss polytope

The girthing polytope. One can prove the non-existence of a polytope by showing its van Oss polytope fails to close.

out

A point is outside a figure, if it lies in the same space that the figure is solid in, but is not part of the figure.

out-vector

A notional vector that radiates from the interior of a polytope. The size of the out-vector is the difference of density over some boundary.

out-vector, transverse

The out-vector as applied to the space of the surface. Since the surface is meant to contain a specific density, the transverse out-vectors must cancel each other out.

Partial Truncate of {3,5,3}.

A non-Wythoffian apeirochora, formed by removing selected vertices of a {3,5,3} so as to reduce every cell into a penatgonal antiprism, and some vertex-figures {5,3}. The vertex figure consist of a dodecahedron with four of its vertices removed, and replaced by triangles...

peak

The single point to which things rise. so appiculation.

Peak

A second-margin. In solid three-dimensional polytopes, this becomes a point, and the raising of the sides to a peak gives the name. The second margin in 4D is an edge. I reserve peak for accumulation to a point. Also smooth polytopes have margins that are not sharp - ie a smooth nanol would not peak in any dimension.

Petrie polygon

A zigzag polygon that is formed by taking n consecutive edges of an n-edge.

Petrie-Coxeter polytope

The family of figures having askew faces and skew vertices: for example, the hexagons of the bitruncated {4;3;4} form a pc{;6%4}. Petrie found the dual pair pc{;6%4} and pc{;4%6}, and Coxeter added pc{;6%6}.

#peton

A 5-edge or a figure bounded by 5-edges (eg a 6-tope)

pi

The number = 3.1415926535897 = 3:16E8 E212 7796 7998. This is defined as the ratio of the circumference to the diameter. This is more a constant of convenience, rather than a "accept-or-else" constant, like the base of the napierian logs.

Picol

A 4-face, or N-4 edge

Piecewise

Being a member of a series that differs by the addition of known elements.

Piecewise Constructable

A polysurtope that can be constructed by adding pairs of surtopes, of adjacent dimensions, from a point.

Piecewise Finite

This is a variety of infinitely dense, where it is possible to complete all the elements that share any given surtope. The tiling of octagons or octagonny are piecewise finite.

Peicewise Sparse

A condition that occurs when there are infinitotope cells in a tiling that covers all space. There is at any point, a cell that has a vertex at any vertex in all-space.

Examples include {oo, 4/2}, {oo,6/2} and {3,5,3,5/2}.

Plane

The orthospace of a line. A plane divides space.

Plane

A 2-flat. The sense here of plane = 2flat is held to proceed geometry, and that its appearance in geometry is to be regarded as a general leader of "wall", "floor", "table-top", that is a solid flat surface that divides all-space.

Plate

An N-1 polytope, or a figure solid in the plane. A plate is a loose thing, though, not necessarily part of a larger thing.

Platonic Figures

A class of regular figures where the transitivity of the flags is due to mirrors. This excludes such exotica as skew and semi-skew regular figures.

Pleat

An margin of small angle, that causes a face to point inwards. Such an angle occurs in the quasilaminates.

Poincare Projection

A projection of hyperbolic space into a disk, such that hyperbolic lines are represented as circles that cross the edge of the disk at right angles. Like the stenographic projection, this one preserves angles between lines. A circle in the poincare projection represents a circle, horocycle, or equidistant curve, depending on whether it shares 0, 1 or 2 points with the edge of the disk. Straight lines have no significance in this projection. [ie they're just circles].

Poly-

Many. It is used to refer to a general member of figures marked with a #, eg polygon is a general member of the series triangle, square, pentagon, hexagon, &c.

Polycell

Many cells. In the context that a cell means a bubble (of a foam), this means many polytopes joined to form a fragment of a foam. For example, the net of a 4D polytope would be a polycell.

Polycell

A polychoron The name polycell preserves treating the surface as a 3D foam on the surface of a sphere.

Polyglomal unit

Unit equal to the content of a Euclidean sphere of unit diameter.

Polyprism unit

unit equal to the content of a unit-edged Euclidean measure-polytope

Polysurtope

A name used for several connected surtopes, but not necessarily a complete figure. So four faces of a dodecahedron would be a polyface.

Polytegmal unit

Unit equal to the content of a unit-diameter Euclidean cross-polytope.

Polytopalotope

A poly-m-topalo-n-tope is a poly-n-tope whose every sur-m-tope is as named. So a pentagonalochoron is a polychoron (4D polytope), whose every surhedron is a pentagon. The twelftychora is an example of this. Poly-m-topes are comparatively rare.

Polytope

A generalised n-dimensional figure. The term was first used by Mrs Stott. The name is comprised of a prefix and suffix stem, defining the mode and the dimensionality of the figure.

Polytope adjectives

Any of the adjectives used to define polytope classes.

```
archimedean   uniform, not prismatic or polytope
catalan       having a symmetry transitive on the faces, equal dha.
compound      having more than one separate surface, in the same side.
johnson       having all surhedra regular, and convex
platonic      regular, where transitions are by mirror-edge
prismatic     being a prism-product
simple        having no holes or intersecting surface.
tegmatic      being a tegum product (eg catalan)
uniform       having a symmetry transitive on the vertices, equal edge
```

Polytope prefix

The prefix defines the mode or meaning of the polytope.

```
anglu-      corner: a vertex of a dodecahedron is three corners.
apeiro-     without end = surface as a tiling in N-1 dimensions
endo-       a polytope marked with all crossings, not just edges.
globlu-     a sphere, represented as an globed-shaped polytope
horo-       the surface follows a zero-curvature or horosphere
lamina-     a polytope bounded by unbounded faces
infinito-   a polytope with infinite faces, and an interior
mulli-      the outline of a polytope in the space it falls in
orthosur-   the corresponding surtope of the dual.
polysur-    many loose surtopes, not forming a complete figure.
pseudo-     the surface follows a negative-curvature or pseudosphere
sesqui-     a second-series polytope, used for different kinds of spaces.
segmento-   an exotic prism, as in a segment of a larger polytope
sur-        surface element, eg, vertex, edge.
```

Polytope Suffix

The suffix defines the dimensionality of the figure. For the etymology of teron to yotton, see 'thousand-rule'.

```
-tope      a general member of this series.
-gon       a  2D polytope  or a  1D tiling
-hedron    a  3D polytope  or a  2D tiling
-choron    a  4D polytope  or a  3D tiling
-teron     a  5D polytope  or a  4D tiling
-peton     a  6D polytope  or a  5D tiling
-exon      a  7D polytope  or a  6D tiling
-zetton    an 8D polytope  or a  7D tiling
-yotton    a  9D polytope  or an 8D tiling
-xenon     a 10D polytope  or a  9D tiling.
```

Presentation

A construction sequence that ends in a given figure. For example, {4,3}, a square prism, and an equilateral rectangle prism are all presentations of the cube. It is not always apparent that different constructions yield the same figure: for example o3m3o4o [the 4D double-cube] is the same as the 24-chora x3o4o3o.

Prism

The product that gives rise to the measure-polytope. It derives from "cut", the sense being that one cuts the assorted bases out of orthogonal all-spaces: for example, a pentagon-hexagon prism might be derived by cutting a pentagon out of the x-y axis, and a hexagon out of the w-z axis. The dual of the prism is the tegum.

The following prefixes have been established for use with prism and tegum. They have individual entries in the gloss.

```
anti-       an exotic prism with dual bases
exoto-      an exotic prism
mod-        a semiate prism where station-sums add to zero mod x
poly-       the prism product of p line segments.
step-       a semiate prism where stations are all equal.
```

#Prism

A polyprism is a measure-polytope. The polyprisms carry special names, and are used as measures: biprism = square, triprism = cube, tetraprism = tesseract.

Prism-circuit

The former name for the runcinate. This reflects the "cycle" of prisms that form most of the faces of a runcinate.

Prism-product

The multiplication of two polytopes by cutting out of space, a section that has each base in turn. For example, a hexagonal prism might be made by cutting a hexagon out of the x-y plane, and a length from the z-line.

prismato-

A term in Bower's naming, this causes the fourth significant node to be marked.

Prismatoreflects

A family of 3 non-Wythoffian Euclidean apeirotopes that occur in every dimension. These are derived from the t-basic, a tiling formed by a simplex and its rectates. The t-basic is a section of the n-cubic, along an axis perpendicular to a diagonal of the cell, and passing through vertices of the cubic.

- The reflects form when one takes a single layer, and use the top and bottom as mirrors. Progression though these mirrors will go through the same kind of rectate cell, rather than progressing through all types. In three dimensions, the vertices fall at the "hexagonal close pack" honeycomb.
- The prismatic forms occur when one set of layers is expanded out, and a slab or prisms is placed there.
- The prismatoreflects occur when one inserts a layer of prisms into a reflect. This has the same vertex figure as the previous, but the prism- edge is now a mirror-edge.

Pseudo-

A prefix meaning 'false', overloaded with the meaning of 'negative curvature'. In the following words, the latter sense is used.

Pseudohedron

The most numerous of uniform tilings are H2 tilings. Each corresponds to a H3 pseudotope, the name pseudohedron covers both meanings. The existence of the 'wrap' operator, and numerous snub operations, like the mobius, marek, and staircase snub, forms an explosive list.

Pseudosphere

A curve equidistant from a flat surface that divides space.

Pseudopoint

A transinfinite point, such as the centre of a pseudosphere. Lines that pass through this point pass perpendicularly through a hyperbolic plane.

Pseudosurtope

A pseudotopic surtope.

Pseudotope

A polytope whose surface generally follows some pseudosphere.

Pseudovertex

A pseudopoint as a vertex.

Pyramid

The general product that produces simplex-polytopes from the vertices. One can replace any number of vertices by a codimensional figure to generalise the product. The pyramid adds an altitude for each base, starting from n=-1.

Pyramid

Some use this word to reflect the exotic prism with two bases, one of which a point.

#Pyramid

The #-1 simplex: the octapyramid being the seven-dimensional simplex polytope. Note that bipyramid is applied to the exotic tower.

Pyramid Altitude

The space orthogonal to all the bases. The transverse at any given point in the altitude is a prism-product of the bases, the fractional size of any given base being the same as the fraction of height to the apex of that base.

Pyramid Apex

A point in the pyramid altitude, representing the vertex of a simplex in the pyramid altitude. At this point, the transverse section consists entirely of a given base at full size.

Pyramid Transverse

The space defined by the cartesian product of the bases: the span of the bases. The projection of the full pyramid into the transverse space is a tegum product of the bases.

q-

Used to refer to figures having the symmetry of the cubic. This has four stations arranged as a square.

q-cubic

The generalised cubic honeycomb. The efficiency is 1/sqrt(2^n), This marks the diagonal of the stations.

q-doublecubic

The body-centred cubic, of efficiency 2/sqrt(2^n) This has all four stations marked.

q-quartercubic

A honeycomb formed by putting a semicubic the vertices and cell-centres of the cubic. This has the symmetry of the cubic, less those parallel to the cubic's margins. The symmetry is reduced to a quarter. The efficiency is 1 q-unit. This marks an edge of the station-square.

q-semicubic

The principle honeycomb, formed by alternating vertices of a cubic. The efficiency is 1/2 q-unit.

q-sesquicubic

A figure formed by a cubic and semicubic. The efficiency is 1.5/sqrt(2^n) q-units.

q-stations

The four points of the q-cubic that preserve the semicubic's symmetry: these are the alternating vertices of a cube, and the centres of the alternating cells. These are arranged in a square, with the cubics appearing as diagonals, and the semicubics as edges.

q-unit

A unit for measuring the efficiencies of honeycombs for packing spheres. 1 q-unit corresponds to putting a sphere of diameter sqrt(2) into a unit measure-polytope. If the honeycomb has an efficiency of n q-units, then a volume of v would contain nv such spheres. The honeycomb that has this efficiency is the q-quartercubic

r#

The square root of #, eg r5 = 2.236067&c = 2:2839 4555 97&c

Radian

An angle subtended by an arc equal to the radian. See also circle.

Radian, #Prismic

This is the common use for solid angles, this marks off a surface of the sphere with a content of 1 #prismic radius. The angle is of course in #+1 dimensions.

Radian, #Tegmal

The angle that marks of a #tegmal radius on the surface of an #+1 dimension sphere. This is smaller than the regular simplex, which is bounded by 1 and sqrt(n/[e=2.718..]) polytegmal radians.

Realm

The infinite extent of "all space", considered as if nothing else existed.

Realm

The name given to 3D space. The shared sense is all-space.

Reciprocation

The process of creating the dual, often by taking the inverse of metric properties. See also inversion-dual. This is more general.

#Rectate

The result of the rectification.

#Rectified

A figure created by joining the centres of the #-edges. The dual of the rectified figure is a surtegmal figure.

Regiment

The 1-army unit, consisting of polytopes that share the same vertices and edges. This grouping occurs quite commonly in the study of uniform polychora.

Regular

A figure whose symmetry is transitive on its flags. The connotations here is that the faces of a regular figure are solid in their spaces and have interiors, even though the figure itself does not. For example, the Petrie-Coxeter and Complex figures are regular.

Rhombus

The term refers to a polygon, being the dual of a rectangle. In three and higher dimensions, there are two valid implementations of the meaning.

- *The rhombus is an equilateral parallelopeid* This resolves into higher dimensions as a polyprism, stretched along one of its long diagonals, or a simplex antitegum. The influence of vectors and lattices has made this the major meaning.
- *The axies of the rhombus bisect at right angles* This makes the rhombus into a tegum. Before the invention of the word tegum, rhombus was sometimes pressed into this use. One should note that the use of "rhombo-" and

"rhombotruncated" by Kepler refers to what in higher dimensions becomes a tegmal product. The expression "surtegum" is recommended for this application.

Rhombotruncate

Like "rhombo-", there is not a rhombus in sight! This expression gets used by G Olshevsky for "cantetruncate", and W Krieger has used it as a version of "omnitruncate". As noted under "rhombus", the rhombus does not appear that often in uniform figures.

ridge

a margin. A margin does not carry the suggestion of sharpness, and in hyperbolic space, some infinitopes can have reflex angles, making the margins into valleys.

Runcinate

A figure formed by radially moving the faces out, without size-change. This produces a series of new faces, formed by the prism product of surtope and its orthosurtope.

s-unit

The angle representing 1/120 of a 4-sphere. This is 120 f units This is the symmetry of the group 4s = [3,3,3].

Second-Series

The second series is used to refer to polytopes in some different kind of space. For example, the points and edges of the Dynkin Symbol are not points and edges of the polytope they describe, and so they carry second series names, such as node and branch.

Segmentatope

[Klitzing] A class of polytopes inspired by taking parallel sections of larger polytopes. The class naturally extends to other types of figures with top and bottom in the same symmetry group, such as cupolae.

The segmentotopes are defined to be any polytopes that obey 3 rules:

- all vertices are on the circum(hyper)sphere
- all vertices are on 2 parallel (hyper)planes
- all edges are of unit length

These differ from exotic simplexes, in that the edges must be uniform. An exotic simplex on elliptic bases and uniform edges is a segmentatope, but exotic simplexes can have unequal edges.

semiate

A process of reducing a cartesian product of several station-systems, according to some modulus property of the ordered stations. For example, one might make a polytope with seven vertices in four dimensions, by numbering the vertices of a heptagon and heptagram in order, and then retaining only those vertices where the vertex-numbers are the same.

The naming convention describes the process well

- The bases are named, so that the sequencing advances the stations advance at the desired speed, eg {15} or {15/2}
- If the modulus of operation is not clear, or different to that of the base, it should be stated here, eg 'penta'
- The name of an relation should then follow. Currently, we have
  - **MOD** The sum of all stations add to zero. This is the way of deriving a generalised semicubic (which has points with an even sum).
  - **STEP** The station values are all equal. This derives the generalised body-centred cubic (ie all 0, all 1, all 2, etc.)
- The next term defines what sort of polytope should arise. Since we are reducing a cartesian product of points, the prism/tegum set applies. A comb-product is regarded as a prism if the vertices are being referred to, and a tegum if the cells are referred to.
  - **PRISM** The vertices of the prism are retained. This generalises the idea of 'half-cube', etc.

- **TEGUM** The faces are stellated. This generalises the idea of a half-cross polytope.

The example referred to in the list might be a {15}{15/2} pentamodprism, a polytope that contains the vertices of 9 pentachora, or 5 bitriangle prisms.

semis-

The usual meaning is to remove alternate (ie odd or even) verticies. The extended meaning is ordered reduction to a single class, when the vertices etc can be allocated to p classes. For example, one can derive the pentachoron from the bipentagonal prism by numbering the vertices of one in pentagon and the other in pentagramic order. Consider only those that add to five. Likewise, the heptepeton or heptapyramid can be formed from the triheptagon prism by advancing the three bases at the rates of 1, 2 and 4. However, the true 1,2,4 semis triheptagon prism would have 49 vertices.

Sesqui-

Properly 'following', taken to mean 1½. Here, it is used to mean a sequence for second-series surtopes, eg sesquihedron = 2D element.

Shortchord

The base of the isosceles triangle formed by two consecutive edges of a triangle, usually taken to be the circumdiameter of the vertex figure. As a number, written as L1, and its square is L2.

Simplex Polytope

The result of a pyramid-power of n+1 points. This polytope has the simplest surface, and the fewest vertices and faces required to form a polytope points. Also called polypyramid.

Skew

A symmetry operation that does central inversion, rather than reflection. This produces the skew or zigzag polygons, such as the Petrie polygons. The dual of a skew polygon is an askew polygon. This has an interior, and has the same rotary-inversion symmetry found in the skew figures. Skew figures appear as the vertex-figures of the Petrie-Coxeter polyhedra.

Smooth

Having the same curvature as the hull of the vertices. For example, a smooth dodecahedron is a sphere marked up as a dodecahedron, regardless of the ambient geometry. So, smooth angles, &c. For example, the smooth angles of the dodecahedron are 120 degrees at the vertex.

Snub

A class of polytope formed by removing alternate vertices of other polytopes.

Snub, Coxeter

A class of snubs formed by selecting alternating vertices of the t{p,2q,r,..} In these cases, the edges of the {p,2q,r,...} can be coherently indexed, and divided in a particular ratio. One of these points gives rise to a uniform snub. This forms a class of non-Wythoffian figures.

- {;3;6} snub trilat = smaller trilat
- {;4;4} sunb quadlat = smaller quadlat
- {;3;4} snub octahedron or icosahedron
- {;3;4,3} snub 24-chora
- {;3;4,3,3} snub {;3;4,3,3}

While Coxeter's rule is fairly good at producing equal-edged figures, there is no guarantee the figure is uniform. s{;3;4,3,4} produces a series of octahedral pyramids as the snubbing face, in place of pentachora.

Snub, Wythoff

A family of non-Wythoff figures

Wythoff's rule of forming snubs of the three dimensional groups, by removing alternating vertices of an omnitruncate. In 3D, this always produces a uniform figure, since it is 3 variables in 3 unknowns. In four and higher, there are more variables than degrees of freedom.

- {;4;4;} snub quadlat = s{;4;4}
- {;6;3;} snub hexlat
- {;3;3;} snub tetrahedron = icosahedron
- {;4;3;} snub cube
- {;5;3;} snub dodecahedron
- {;3;3;A;} snub 24-chora = s{;3;4,3}
- {;3;3;A;B;} snub {;3;4,3,3}

Solid

Having an orthospace of a point. No line may be drawn in the space that is orthogonal to every line drawn inside the solid. For example, in 3-space, no line may be drawn orthogonal to a cube, but a line may be drawn orthogonal to a square.

When a figure is solid in some space, there is some radius, that any point lieing in the space, and less than that radius, lies also in the interior of the figure. A hexagon might be described as "solid" in a plane in 3D, even though it is not a 3D solid.

#-Space

The extent of points, having # dimensions. Without the numeral, the implication is the dimension that makes the desired results solid.

Span

A set formed by multiplying a set, eg Z, F, over a set of vectors or numbers.

Sphere

The solid formed by the points equidistant from a fixed centre.

#-Sphere

The set of points in #space equidistant from a common centre.

Sphere

A 3-sphere. I use the terms based on glongotope and #-glome to form spheres between the circle and the solid sphere.

Square

The regular tetragon. As a measure, the content of a Euclidean square is implied.

Square

Some authors require a square to have right-angles. In practice, any {4} will yield a right-angled tetragon when projected onto a chordal surface containing the vertices.

Staircase Snub

A kind of modified mobius snub found in uniform pseudohedra. In the diagrams below, each different number represents a different edge class.

```
x 4 x 2 x 4 x 2 x     o 2 x 3 x 2 o 1 o
3   1   3   1   3     2   1   2   3   2
x 2 x 4 x 2 x 4 x     x 3 x 2 o 1 o 2 x
1   3   1   3   1     1   2   3   2   1
x 4 x 2 x 4 x 2 x     x 2 o 1 o 2 x 3 x
3   1   3   1   3     2   3   2   1   2
x 2 x 4 x 2 x 4 x     o 1 o 2 x 3 x 2 o

  Normal snub          Staircase snub
```

An example was presented in Marek Ctrnack's enumeration of the uniform pseudochora of the vertex-type ppppq, where the edge 3 is wrapped from a digon to a triangle.

Station

[standing point] If polytopes represent the geometric integers, then stations would be the modulus classes. The shape of a station is that of the vertices matching the same modulus. The interaction of stations have great importance on compounds.

For example, the modulus of 6 over a twelfty-gon produces 6 stations each in the shape of a 20-gon. This 20-gon can settle in the available 120 points in 6 different standing points or stations.

The term originated in tilings, where the fundamental tiling of the t, q and y groups could stand at each of a number of different points of the fundamental region.

#-Stellation

The extensions of the #-edge until the figure closes again. As a general process, the n-stellation extends the n-edges, so 2-s~ gives greatening, 3-s~ gives grand figures.

Stepprism

A semiated prism, retaining vertices where all ordered-stations are equal, relevant to some modulus: see 'semiate', 'modprism'.

Steptegum

A semiated tegum, retaining faces where all ordered-stations are equal, relevant some modulus: see semiate, modtegum.

Stereographic projection

A projection that preserves isocurves and angles. Such projections are widely used, and can be used as 'pocket geometries' in all geometries. The Spheric form makes an isocurve that pass through diametric points of the equator "straight". The Euclidean form makes any isocurve that passes through a fixed point (point at zero) straight. When this is done on a Euclidean plane, the result is the same as inversion, although the result can be replicated in any other space. The hyperbolic makes an isocurve that passes through a fixed circle (horizon) at right angles, as straight. This is the poincare projection.

Stott Construction

The creation of polytopes by the radial expansion, with no change of size, of the surtopes of a tiny polytope. For example, if the vertices of a cube were radially expanded, the figure would result in a larger cube. If the faces were radially expanded, the figure results in a rhombicuboctahedron.

Stott Matrix

A matrix formed by the dot-product of Stott vectors. Applying the matrix-norm to a Stott-vector gives the circumdiameter of the polytope it represents.

Stott Vector

The Stott construction can be regarded as moving the vertex parallel to a given axis of the reflective region. This preserves the size of everything that has the same symmetry as the axis, and creates new sizes for the other elements. This table shows the stott vectors for the octahedral and icosahedral groups. When added in the manner indicated, these produce a figure of edge 2.

```
     V         E         H          Vectors making uniform fig

   1.0.0     0.1.1     r2.r2.r2       v      octa    icosa
                                      e      CO      ID
   f.1.0    2f.0.0     f2. 0. 1       h      cube    dodeca
   f.1.0    f2.1.f     f2. 0. 1      v+e     t oct   t ico
   f.1.0    f2.1.f      f. f. f      e+h     t cub   t dod
   f.1.0    f.f2.1      f. f. f      v+e     srCO    srID
   f.1.0    f.f2.1      1. 0. f2    v+e+g    grCO    grID
```

Strombotope
: An antitegum.

Strombopolytope
: A figure bounded by antitegums.

Strombiate
: To replace each face of a figure by an antitegum, radial from the centre. The effect is to create antitegums axial on the face-vertex axis, with a section being the vertex-figure of the face. This is the join operator in Conway's notation. As a surface operation, this erects an antitegum axial on the line from the vertex to the face-centre, and in section the vertex-figure of the face.

Strombus
: A quadrilateral formed by reflecting a triangle in one of its sides.

Surcell
: The parts of all-space in a honeycomb. The surface of an apeirotope when it has been reduced to a surface itself.

Surcell
: 4d mullisurface. The sense here is by analogy from face/surface + cell/surcell.

Surchoron
: A 3-dimensional surtope. In four dimensions, this is called a face, since a 4D polytope is bounded by surchora.

Surhedron
: A 2-dimensional surtope. In 3D, this is often called a face, since surhedra bound the 3D polytope.

Surface
: A surface of a solid properly divides the space it is solid in, into points in and out of the solid. However, the notion generalises by defining a surface as a connected space that piecewise divides space.

Surface
: The 3D mullisurface. The surface of what appear to be highly self-intersecting polytopes do not actually intersect. One can not "see" more than one surface-plane or flag at once. The drawings of polytopes are usually of the endocell.

surround
: The sense of this, is to occupy all points in the solid space of a given figure. This makes a second "surface" for the polytope, in a sense.

#Surtegmal
: The dual of a #rectate, with faces being tegums of the surtope and orthosurtope that is represented by a vertex of the #rectate. The operation of #surtegmating makes the #edge of the dual, and the #-margin of the figure into the bases of a tegum.

Surtope
: Any of the surface polytopes of a polytope, ie the general member of the family vertex, edge, ..., margin, face, cell.

Surtope

[Olshevsky] The sense appears to be mullicell. Since sur has been defined in terms of the re-enterant surface, the exposed-only version is allocated a new prefix under mulli-

Surtope excess

The evaluation of the surtope polynomial, including noids and content, evaluated with a=-1. This in 3D counts the number of holes: eg for a bi-hexagon torus = $a3+36a^2+72a+36+1/a = -2$. This represents the two holes of the torus in 3D: the hole at zero and the hole at infinity. For a polytope like the {5,5/2} = $a3+12a^2+30a^1+ 12+1/a = -8$. These eight holes might be made by removing the 20 faces of the icosahedron, and restoring 12 faces as the sectional pentagon. In higher dimensions, the surtope excess is not always connected to the holes, since the hole polynomial may evaluate to zero in even dimensions.

Surtope figure

The arrangement of higher-order surtopes around a given surtope. Thus, an n+m surtope is represented as a m-1-tope in the n-surtope figure. This is a generalisation of the vertex figure. The margin-figure is simply a polygon representing the different faces incident on the margin. For any of Gosset's figures, this is a r2:r2:1 triangle, the r2 being the diagonal of the polytegum or cross polytope, and the 1 being the opposite edge of the polypyramid or simplex.

Surtope polynomial

An expression of the count of surtopes, where the n-edge is represented by the nth power of some value a, eg cube = $6a^2+12^1+8a$. Euler showed that, when a=-1, this evaluates to 2 for odd dimensions, and 0 for even dimensions. This equation can be made to add to 0 for all dimensions, by adding terms for the content (eg 1a3), and the dual of the content, or noid (1/a). The surtope polynomial is also called the Euler characteristic.

t-

A prefix used to define a class of honeycombs having the "t" symmetry. This is the one formed on treating the edges of a simplex as vectors. This has n+1 stations, arranged in a polygon.

t-basic

A lattice formed by allowing translations of the edges of a simplex. The vertex-figure is a runcinated simplex. The efficiency in q-units is 1/sqrt(n+1) The cells of this is the simplex and its rectates. This is cut by planes parallel to the faces of a simplex, and spaced at the height of a simplex. Because of this, it gives rise to a series of laminate non-Wythoffian figures, being the prismatoreflects.

t-catseye

The tiling of the rhombotopes clustered together to resemble a cube-corner in projection. The name catseye comes from reflectors that reflect light in the direction it comes from, this is done by using a section of cubes in the plane x1+....+xn = constant. The dual of this is the t-truncate.

t-diamond

A positioning of spheres on two consecutive stations of the t group. Around each point is arranged a simplex of the opposite "colour", in upwards and downwards pointing positions. In 3D, this represents the position of atoms in a diamond lattice.

t-rhombic

This is the dual of the t-basic. The cell is the strombiated simplex, formed by replacing each face of a simplex with an obtuse rhombotope or parallelotope.

t-stations

Any of the vertices of the fundamental region of the simplex. In terms of the 60° rhombotope, the t-stations divide the long diagonal symmetrically into N+1 points. These are the cell centres.

t-truncate

A Wythoff construction based on a pair of consecutive marked nodes. This is a tiling of the simplex and its polytruncates.

#Tegmal

used as a measure equal of the content of a #tegum. This may be applied to radians as well.

Tegum

The product that produces the cross polytope. This generalises to one that covers the products placed orthogonal to each other.

tegum takes some prefix-names as well, the list being as follows. These are also glossed individually.

```
anti-     the exotic tegum of duals
exoto-    an exotic tegum
mod-      the semiate stellation based on modulo-sums
poly-     the tegum-product of line-segments.
step-     the semiate stellation based on modulo-equalities
sur-      a surface tegum, such as a rectate-dual
```

#Tegum

The tegum of # equal lines, giving the cross polytope in # dimensions. When the lines are unit, the #tegum may be used as a measure, to which the tegum and pyramid products are coherent. 1 #prism = #! #tegums.

#teron

A 4-edge, or a figure bounded by 4-edges.

Tesseract

The usual name for the tetraprism {4,3,3}

tesseract

Sometimes restricted to right-angled {4,3,3} only. Also called octachoron.

tetra-

A prefix meaning four or fourfold, in the senses of #.

Thousand-rule

The use of metric prefixes to represent polytopes. In this series, the edge is rated as 1000, and the surtope-name gets its name from the metric prefix for 1000^n

```
tera-  = 1e12    -teron  = 4d tiling or 5d polytope
peta-  = 1e15    -peton  = 5d tiling or 6d polytope
exo-   = 1e18    -exon   = 6d tiling or 7d polytope
zetto- = 1e21    -zetton = 7d tiling or 8d polytope
yotto- = 1e24    -yotton = 8d tiling or 9d polytope
```

The series continues downwards, to name the assorted margins, based on the submultiple prefixes, as if the base form is "face", with milliface being margin. No name exists as yet for the second-margin. N-2 N-3 N-4 N-5 N-6 N-7 N-8 N-9 milli- micro- nano- pico- femto- atto- zepto- yocto- margin nanol picol femtol attol zeptol yoctol This series of names are meant to stand in apposition to numbers in most languages, without confusion. In accordance with -hedron and -choron, they take the same plural: that is -on becomes -a.

Thousands-rule (Bowers)

Jonathan Bowers extended the thousands-rule, in the process, inventing a new series of numbers. Such numbers preserve the assorted Indoeuropean irregularities. The names here refer to the N-d tiling, or the N+1 polytope.

	0	10	20	30	decades		centuries	
0		dakon	icon	tracon				
1	gon	hendakon	ikenon	trakenon			100	hoton
2	hedron	dokon	icodon	tracodon			200	doton
3	choron	tradakon	ictron	tracotron	30	tracon	300	troton
4	teron	teradakon	icteron	tracoteron	40	teracon	400	teroton
5	peton	petadakon	icpeton		50	petacon	500	petaton
6	exon	exdakon	icexon		60	exacon	600	exoton
7	zettin	zettadakon			70	zettacon	700	zettaton
8	yotton	yottadakon			80	yottacon	800	yottaton
9	xennon	xendakon			90	xencon	900	xennoton

Examples: 123 hotictron 999 xennotaxennocoxennon

The series extends into the thousands as well.

Time

The role of time in geometry is to focus on a sequence of diverse events as if they were snapshots of something in motion. One must be aware that what is presented as the same thing is in fact different but similar things, and the desired equality must be proven.

#-tope

A figure that is solid in # dimensions, and bounded by lesser surtopes. This is the general #-dimensional polytope.

torus

A product of two figures that is inscribed in a torus. The first named figure is the hole at zero, the second the hole at infinity. The surtope equation is the same as comb-product. A pentagon-dodecahedron torus is made by bending a stack of dodecahedral prisms into a pentagon. This produces a circle that is interior to the torus. On the other hand, a dodecahedron-pentagon torus is made from the stack of dodecahedra prisms, by rolling the outer skin down as one might take off a skirt or socks. The sphere that surrounds the dodecahedron now becomes interior.

#-torus

A polytope that contains a "solid" hole. That is, the space where the non-vanishing loop is drawn, is solid in the dimension of the polytope. Such is usually a hole at infinity or a hole at zero.

Transverse symmetry

The symmetry orthogonal to the altitude in an exotic simplex or pyramid. In a pentagonal antiprism, the bases have pentagonal symmetry, and this is the transverse symmetry.

Tri-

A prefix meaning three or thrice, eg in the meaning of #.

Trigional groups

The Euclidean groups that have mirrors set at only 90° and 60° only. These can be built up from layers of the t-basic, by advancing one (t), two (q) and three (y) stations. While other numbers are possible, eg 0, 4, these do not produce any new groups. The version on 0 simply is a comb product of the apeirogon and the t-basic, and the 4 and higher dimensions are alternate presentations. The primary lattice, built on a base of n, and advancing x stations has $x^2+(2-x)n$ stations, the efficiency being 1/sqrt(s) in q-units.

true length

The length measured along a straight line. In practice, chordal length is used.

true space

A special kind of all-space which is generally regarded as encompassing all-space. Eg, for the Earth, all-space is taken as the 2D spheric surface, and true-space is the 3D Euclidean space. When considering the apeirotopes, the true-space is taken to be co-incident with the smooth surface, and the all-space is taken to have some kind of interior.

#Truncate

The operator that removes vertices through a vertex-bevel. The truncate is usually the portion until the edges are exhausted by the beveling. In regular figures, the #truncate means the planes have removed all the #-1 edges, but not the #-edge.

Uniform

A figure with a symmetry transitive on the vertices. Unlike regular figures, there is no restriction that the faces of a uniform figure need to be solid.

Uniform-face

The figures whose symmetry is transitive on the faces. These are the duals of the uniform figures. Uniform-faced figures are the duals of the uniform figures.

Vertex

A 0-dimensional or point surtope.

Vertex figure

The arrangement of surtopes around a vertex, as the vertex figure becomes vanishingly small.

Vertex Node

The vertex node is a node in the Dynkin symbol, to which all marked nodes are notionally connected to. The act of making these connections real converts the figure represented in the dynkin symbol into the vertex figure of a new polytope.

A polytope may have any number of vertex nodes, such becomes an exotic pyramid. The act of converting marked nodes into vertex-nodes is the manner of generating a vertex-figure.

{w#}

A designation of a polygon by the square of the shortchord L2. In the cases where the number of sides is not easily described, this is the most consistent description. eg one might write the pentagon as {w2.61803398875}. Stars are designated in the usual way, for example, the pentagon can be written as {10/2} or {w3.61803398875d2}. In the case of infinigons, this is the only designation that may be used to denote these.

Wall

An N-1 figure that divides.

wrap

A process, usually in H2, of increasing the repetition of segments. The operation is the product that produces the infinite class of polygons, and in pseudohedra, produces all sorts of multiplications. For example, one may wrap a vertex figure, to make a second tiling with twice as many faces at each vertex, the vertex figure being a double-wrap of the original vertex figure.

One can wrap any sort of polygon, including snub cells, glides, etc.

Wythoff construction.

The mirror-edge construction, and the reduction of these by removal of alternating vertices (snub). These produces all but one of the uniform polytopes. The process can produce a guaranteed uniform figure for every combination of mirrors if the symmetry group is a simplex.

y-

The prefix used to refer to Gosset honeycombs. These have 9-n stations, marked as a polygon. The four dimensional case is the same as the t- honeycomb, but the stations are in pentagrammic order.

y-gosset

The primary gosset-honeycomb, having an efficiency of 1/sqrt(9-n) q-units. In 3-8 dimensions, these are: triangle-prisms, 4D=t-basic. 5D q-semicubic, 6D 4B1 = 2_22 , 7D 6C= 3_31, 8D 7B= 5_21.

y-station

The 9-N vertices of the fundamental region, where the point reflects to form the y-gosset. The order of these is polygonal. For 2D, this is the centre of a triangle of edge 1,r2,r2, for the 3D, the hexagon is a zigzag around the squares of a triangular prism, for 4D, this is the pentagrammic order of the t-stations.

Yickle

[spear, as in ice-yickle = icicle - ice-sphere] A figure with unbounded margins, which may be used in reflections. The bt{3;4;3,8} has such yickles, the reflections in these fill all space with bt{3;4;3}.

#yoctol

A 8-face or N-8 edge

#yotton

An 8-edge, or figures bounded by 8-edges, ie a 9-tope.

Z#

The Z span of chords of a {#}. Z1=Z2=Z3 = integers. The set Z5 is the integer span of the chords of a pentagon, ie z_1 + z_2 x fi. Likewise, Z7 is z_1 + ha x z_2 + hbx z_3

Z-span

The integral span over a set of vectors or numbers.

zero, at

The result of inversion of at infinity. A isocurve that contains the point at zero is flat in the inversion-geometry. Holes at zero are interior to the solid (but not cavities).

zero-curvature

An isocurve having Euclidean geometry. Unlike other curves, this retains the same curvature on dilation.

#zeptol

A 7-face, or N-7 edge.

#zetton

A mounted 7D surtope. The figure described is by the prefix, a polyzetton is an 8D polytope, an apeirozetton a 7D tiling.

zonotope

A polytope where every surtope has central symmetry. Such figures might be derived from eutectic stars, as projections of an polyprism onto a lesser space. For example, the rhombic dodecahedron is a zonotope, because its surhedra all have central symmetry. It can be viewed as a projection of the tesseract or tetraprism onto a three-dimensional space. The pentagonal dodecahedron is not a zonotope, because pentagons do not have centre of symmetry.

ZZ

The sum of sets Z#, or the span of all chords of all polygons.
